Catenary — Or How a Mathematician Anchors: A Digest for $A_{\text{eff}} = 8\,\text{m}^2$

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This note is intended to provide a brief digest of my longer work on anchoring, in German, found at `https://trimaran-san.de/die-kettenkurve-oder-wie-ein-mathematiker-ankert/`, where the underlying models are explained in detail and a full derivation of all equations is given. Also, this note is specifically geared towards the parameters of a vessel with a windage area of $A_{\text{eff}} = 8\,\text{m}^2$.

Essentially, the intention is to work out the minimally required anchor chain length that is needed to maintain the chain at a horizontal angle at the anchor whilst the wind and other forces are pulling at it. Our results do not deal with whether the anchor is strong enough, only how to give the anchor the best possible chance to have a good grip by having the chain pull at it only in a horizontal fashion — assuming the seabed is horizontal. For references to earlier work on this topic please consult the long German version.

Two mechanisms are considered here:

- **Static anchoring**, which describes a force acting on the vessel, be it wind (also in gusts) or current.
- **Dynamic anchoring**, which describes bursts of energy being transferred to the vessel head–on, like by huge waves and swell.

We find that static anchoring is dominant when anchoring in deep water, whilst dynamic anchoring can be dominant in shallow water, depending on which direction the energy burst is coming from. In particular in shallow water the load on the anchor can then exceed its specifications, in which case it is vital to move to deeper water. As we will see later, simply adding more chain in such a situation is inadequate.

The catenary describing the shape an anchor chain takes is given by

$$y(x) = a\left[\cosh\left(\frac{x}{a}\right) - 1\right], \tag{1}$$

Table 1. Wind pressure p_{stau} on an area with $c_p = 1$ at 20°C at sea level. Parameter a is calculated for 6, 8, 10 and 12 mm chains, with windage area $A_{\text{eff}} = 8\,\text{m}^2$. Wind load F_W at anchor according to Eq. (4).

Max wind strength			Wind pressure in N/m^2	Wind load F_W in kp	a in m for $A_{\text{eff}} = 8\,\text{m}^2$			
in BFT	in Knots	in m/s			6 mm	8 mm	10 mm	12 mm
0			0.03					
1	3	1.5	1.4	1.14	1.63	0.936	0.571	0.405
2	6	3.3	6.6	5.38	7.69	4.41	2.69	1.91
3	10	5.5	17.6	14.4	20.5	11.8	7.18	5.09
4	15	7.9	37.6	30.7	43.8	25.1	15.3	10.9
5	21	10.7	68.9	56.2	80.3	46.1	28.1	19.9
6	27	13.8	114.6	93.5	133.5	76.6	46.7	33.1
7	33	17.1	176	143.5	205.0	117.6	71.8	50.9
8	40	20.7	258	210.4	300.6	172.5	105.2	74.6
9	47	24.4	367	299.3	427.6	245.3	149.6	106.1
10	55	28.4	499	406.9	581.3	333.6	203.5	144.3

where cosh() is the hyperbolic cosine function, and the parameter a is given by the ratio of the wind force F_W acting on the vessel, to the weight (in water) of the chain per metre, mg,

$$a = \frac{F_W}{mg}. \tag{2}$$

Here g is the gravitational constant $9.81\,\mathrm{m/s^2}$. DIN 766 chain weight in water is: 5 mm: $m = 0.44\,\mathrm{kp/m}$; 6 mm: $m = 0.7\,\mathrm{kp/m}$; 8 mm: $m = 1.22\,\mathrm{kp/m}$; 10 mm: $m = 2\,\mathrm{kp/m}$; 12 mm: $m = 2.82\,\mathrm{kp/m}$; 13 mm: $m = 3.4\,\mathrm{kp/m}$; 16 mm: $m = 5.1\,\mathrm{kp/m}$.

For a chain like this, attaching horizontally to the anchor and then immediately curving upwards — and neglecting swell for the moment — its length L until it reaches the water surface is given by

$$L_{\mathrm{static}}(Y) = \sqrt{Y(Y+2a)}, \tag{3}$$

where Y is the water depth at the position of the anchor.

Clearly, the wind force F_W acting on the vessel will depend on the strength of the wind, expressed as p_{stau}, as well as on the cross–section shape A of the vessel facing the wind. This shape gets modified by a factor $c_p < 1$ describing how streamlined it is, resulting in an effective windage area $A_{\mathrm{eff}} = c_p A$.[1] This yields

$$F_W = c_p A\, p_{\mathrm{stau}} = A_{\mathrm{eff}}\, p_{\mathrm{stau}} = A_{\mathrm{eff}}\; 0.602 \frac{\mathrm{kg}}{\mathrm{m}^3}\; w^2, \tag{4}$$

where w is the wind velocity in m/s and A_{eff} is measured in m^2.

For this digest we assume a windage area of $A_{\mathrm{eff}} \approx 8\,\mathrm{m}^2$. In Table 1 we have listed the resulting wind load F_W as well as the values of parameter a for 6, 8 and 10 mm chains as they might be used in such a setting. When using this table one should **always pick the wind strength during a gust** and not the mean wind strength so as to be on the safe side. For each Beaufort (BFT) wind range the values refer to its upper limit.

Eq. (3) establishes the chain length needed for static anchoring, when only wind and current forces are pulling at the vessel. For dynamic anchoring, in addition, a temporary energy burst ΔE needs to be absorbed by the vessel and its anchor gear. In our model we assume this to happen by raising the anchor chain further off the seabed — whilst still maintaining a catenary shape — and thereby storing additional potential energy in the chain. Working out the maths yields for the required minimal chain length[2]

$$\tilde{L} \approx L_{\mathrm{static}} + \sqrt{\frac{3 L_{\mathrm{static}} \Delta E_{\mathrm{pot}}}{mgY\left(1 + \frac{3Y}{5(Y+2a)}\right)}}. \tag{5}$$

As before, the first term describes static anchoring, whilst the second term relates to dynamic anchoring. For $Y \to 0$ this latter term diverges as $Y^{-1/2}$ and so, clearly, it will become dominant at some point when trying to anchor in very shallow water. For large values of Y this term does not contribute a lot and thus static anchoring will be dominant.

As long as the chain pulls only horizontally, the force acting on the anchor is given by

$$F_A = mg\frac{\tilde{L}^2 - Y^2}{2Y}. \tag{6}$$

From this we see that for a chain of fixed m the load on the anchor depends solely on two variables: The water depth Y at the anchor and the chain length $\tilde{L}$ that is actually above the seabed and not

[1] See also `https://de.wikipedia.org/wiki/Winddruck`.

[2] Eqs. (5) and (7) depend on only four parameters: p_{stau}, A_{eff}/m, $\Delta E_{\mathrm{pot}}/m$ and $F_{A\,\max}/m$. Scaling, a Mathematician's delight! Example: The graph for an 8 mm chain with $A_{\mathrm{eff}} = 8\,\mathrm{m}^2$, $\Delta E = 100\,\mathrm{J}$ and $F_{A\,\max} = 400\,\mathrm{kp}$ will be the same as for a 6 mm chain with $A_{\mathrm{eff}} = 4.59\,\mathrm{m}^2$, $\Delta E = 57.4\,\mathrm{J}$ and $F_{A\,\max} = 229.5\,\mathrm{kp}$, or a 10 mm chain with $A_{\mathrm{eff}} = 13.11\,\mathrm{m}^2$, $\Delta E = 163.9\,\mathrm{J}$ and $F_{A\,\max} = 655.7\,\mathrm{kp}$.

lying around. Consequently, should this load on the anchor be too large — either because the seabed is unsuitable and thus holding power is poor, or because the anchor is approaching its limits — it will not help to increase the chain length. It would have no effect on $\tilde{L}$, and hence no effect on F_A. The only course of action is to increase Y and relocate to an anchoring position in deeper water! However, this only works, of course, as long as dynamic anchoring is relevant. It is clearly not possible to reduce F_A below the wind force F_W of the static anchoring. Once this limit is reached, only an additional anchor will help.

If we do know the maximum holding power $F_{A\,\max}$ of the anchor, like from a field trial or so, we can work out the associated maximum chain length $\tilde{L}_{\max}$ that can just be tolerated as a result of Eq. (5) as[3]

$$\tilde{L}_{\max} = \sqrt{Y\left(Y + \frac{2F_{A\,\max}}{mg}\right)}. \tag{7}$$

Obviously, we can still lay out more chain than this limit, as long as this extra chain is lying on the seabed. No harm in doing that, on the contrary.

The force acting on the bow is simply given by the law of Pythagoras as

$$F_{\text{bow}} = mg\sqrt{\tilde{L}^2 + \tilde{a}^2} = mg\frac{\tilde{L}^2 + Y^2}{2Y} = F_A + mgY. \tag{8}$$

Now, how large is this energy burst ΔE that needs to be absorbed by the vessel and its anchor gear? This energy is coming from, e.g., a large swell and will initially get the vessel moving a bit, resulting in a kinetic energy of

$$E_{\text{kin}} = \frac{1}{2}Mv^2 = \frac{1}{2}M(v_\parallel^2 + v_\perp^2), \tag{9}$$

where M is the mass of the vessel and v its velocity. This velocity vector is resolved into a component parallel to the anchor chain, $v_\parallel$, and a component perpendicular to it, $v_\perp$. Obviously, for dynamic anchoring only $v_\parallel$ is relevant — and only when pointing *away* from the anchor, thereby adding strain to the chain. For a 9 metric tonnes vessel and an assumed velocity of $v_\parallel = 0.65\,\text{kn}$, we arrive at $\Delta E = 500\,\text{J}$. For $\Delta E = 25\,\text{J}$ the velocity would have to be as low as 0.14 kn. Just observing the Speed Over Ground (SOG) whilst at anchor suggests that it is not realistic to assume a ΔE much smaller than that in just about any scenario. On the other hand, $\Delta E = 800\,\text{J}$ corresponds to $v_\parallel = 0.82\,\text{kn}$, which is also not that large and still does not describe extremely difficult anchoring scenarios.

In the following we give examples of how much chain is needed when — according to our model — this energy burst is absorbed by temporarily raising the anchor chain higher above the seabed and thereby increasing its potential energy. In Figs. 1 to 12 we have plotted $\tilde{L}(Y)$ (red solid lines) for various chains and various amounts of $\Delta E = 6.25\,\text{J}, 12.5\,\text{J}, 25\,\text{J}, 50\,\text{J}, 100\,\text{J}, 200\,\text{J}, 400\,\text{J}, 800\,\text{J}, 1600\,\text{J}$. From about 4 BFT the agreement between the approximation of Eq. (5) and the numerically exact solution is within a few percent.

To give an idea of how the maximum holding power of the anchor limits our options via Eq. (7), we have plotted as a solid black line the limiting curve $\tilde{L}_{\max}$ for an assumed maximum holding power of $F_{A\,\max} = 400\,\text{kp}$. Anything above this curve would exceed the allowed load on the anchor and thus is taboo. For further illustration, we have also plotted the $F_{A\,\max} = 150\,\text{kp}$, 250 kp and 600 kp lines.

It is obvious that as the wind strength or ΔE increases the curves move up, requiring more anchor chain to be laid out. This is expected. What is striking in particular, though, is that for anchoring in shallow water things change quite dramatically. The required chain length does not decrease monotonically as one tries to anchor in more and more shallow water. In fact, eventually it will increase again! It is very easy for the anchor load to become too high in such a situation, and the only way to reduce

[3] Eq. (7) is the same curve as found for purely static anchoring with $a_{\max} = F_{A\,\max}/(mg)$, reminding us of the fact that for static anchoring the load on the anchor is the wind force F_W only and thus is independent of the water depth Y.

this load is to leave the shallow water and use a little more chain. It is not possible to anchor safely in extremely shallow water when the anchor chain has to absorb substantial energy bursts because of large swells head–on. Using anything less than 19 metres of an 8 mm chain also seems impossible in just about any scenario.

The physics behind this observation is that the kinetic energy introduced by the swell will have to be absorbed by an increase in the potential energy of the chain. Now, if the chain is too short, or is long but hanging too horizontal in shallow waters, it is simply unable do that.

But to be clear, what matters for dynamic anchoring is additional energy in the form of, e.g., swell *coming head–on*. Typically, this will only be the case when we are forced to anchor with onshore wind. Only in such a situation may dynamic anchoring be relevant. With offshore wind it is much less likely to experience swell head–on. Exceptions might be ferries and other large vessels approaching a harbour and one being at the sideline.

When comparing graphs for different chains one does find that the thicker a chain is the shorter it may be to achieve safe anchoring. This is not surprising, but does come at a cost: A 6 mm chain of 100 metres length weights the same as 57.1 metres of an 8 mm chain, or 34.8 metres of a 10 mm chain. Now, when we examine the three graphs for $\Delta E = 200\,\mathrm{J}$, then with 100 metres of 6 mm one finds a sweet spot to anchor safely at 7 BFT, whilst still obeying the (assumed) load limit for the anchor. However, for 57.1 metres of 8 mm this sweat spot is found at 6 BFT, and for 34.8 metres of 10 mm one can only anchor safely up to 5 BFT. This finding is even more pronounced when static anchoring is dominant: We take the same three example chains, but now with $\Delta E = 6.25\,\mathrm{J}$ only, and we assume gusts up to 8 BFT. From the top graph of Fig. 1 one reads that with a 6 mm chain 100 metres long one may anchor at water depths not exceeding $Y = 14.9\,\mathrm{m}$. With Fig. 5 for the 8 mm chain and Fig. 9 for the 10 mm chain this limit is reduced to $Y = 8.2\,\mathrm{m}$ and $Y = 4.8\,\mathrm{m}$, respectively. This means that for the same total weight of chain one has considerably more chances of finding a suitable anchorage spot when using a thinner chain. This is particularly important in areas with strong tides. The thicker chain's main advantage — apart from it being stronger and passing less swell–induced load to the anchor — is to reduce the swinging circle around the anchor. It depends on one's sailing area whether this is more important or the ability to anchor also in deeper or tidal waters.

Obviously, when using a chain as thin as 6 mm one might want to choose a better material such as, e.g., Duplex.

For all calculations we assumed the seabed to be horizontal at the anchor. In fact, when working out the potential energy of the anchor chain we assumed the seabed to be horizontal all the way between anchor and vessel. When the seabed is steadily rising towards the vessel then less chain will be needed for static anchoring, but more so for dynamic anchoring, and vice versa.

Also, all calculations were done with the anchor chain ending at the water surface. So, add a few metres to the bow roller for the anchor chain or — more accurately — measure the water depth from this roller. Adding this length to your vessel's draft constitutes the inaccessible region in all graphs.

Finally, using a long thick anchor chain bridle which can absorb a good part of the energy — best assisted by additional good rubber springs — will go a long way to help keep the effects of dynamic anchoring in check. Good bridles of more than 10 metres can easily absorb 1000 J and more, allowing one to anchor in somewhat more shallow water again. Also for monohulls one should use as long as possible elastic snubber lines for the chain and add rubber springs or the like. In Figs. 13 to 24 we have plotted the results for two different snubbers, one being nominally stretched to 25 cm at 8 BFT and no swell, the other to just 10 cm. These elongations may appear small, but it should be noted that in a strong swell the loads are much higher and will result in the snubber being stretched well beyond one metre in shallow water, where they are most effective, thereby likely overloading the snubber.

This note shows that the old recommendation of using 3, 4 or 5 times the water depth as chain length is not adequate. In deep water it often overestimates the chain length required, thereby possibly depriving you of nice anchorage locations, whilst in shallow water it seriously underestimates it, putting everybody anchoring in your neighbourhood at risk — as well as you and your vessel.

The author would like to thank Bjarne Knudsen, Harald Melwisch and Gerhard Ohm for many most useful discussions on this topic.

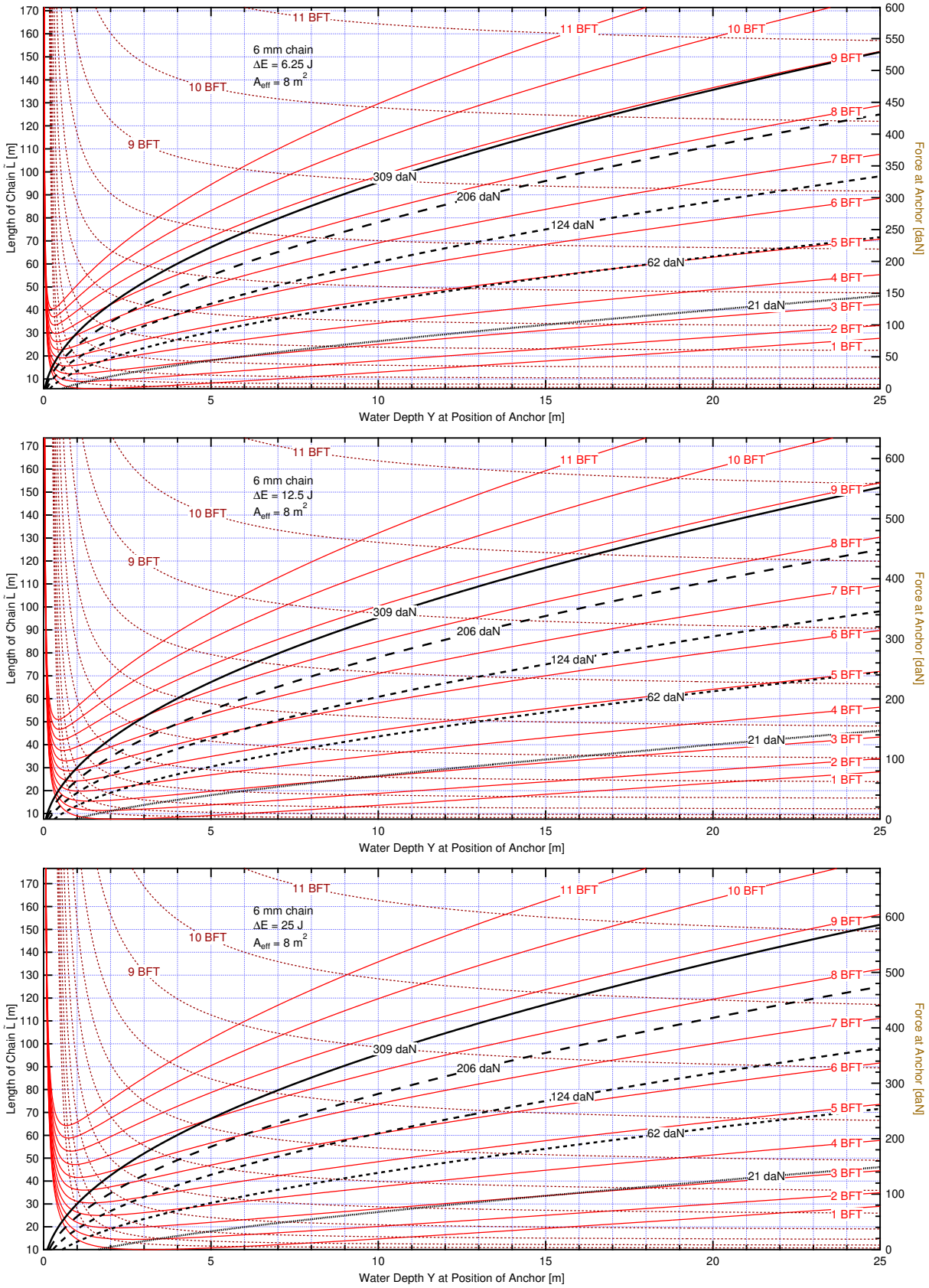

Fig. 1. Required chain length $\tilde{L}$ as function of anchor depth Y and wind strength for 6 mm chain with $A_{\text{eff}} = 8\,\text{m}^2$, $\Delta E = 6.25\,\text{J}$, $12.5\,\text{J}$, $25\,\text{J}$. Anchor load (brown) to the right. Black lines represent various *assumed* limits to anchor load.

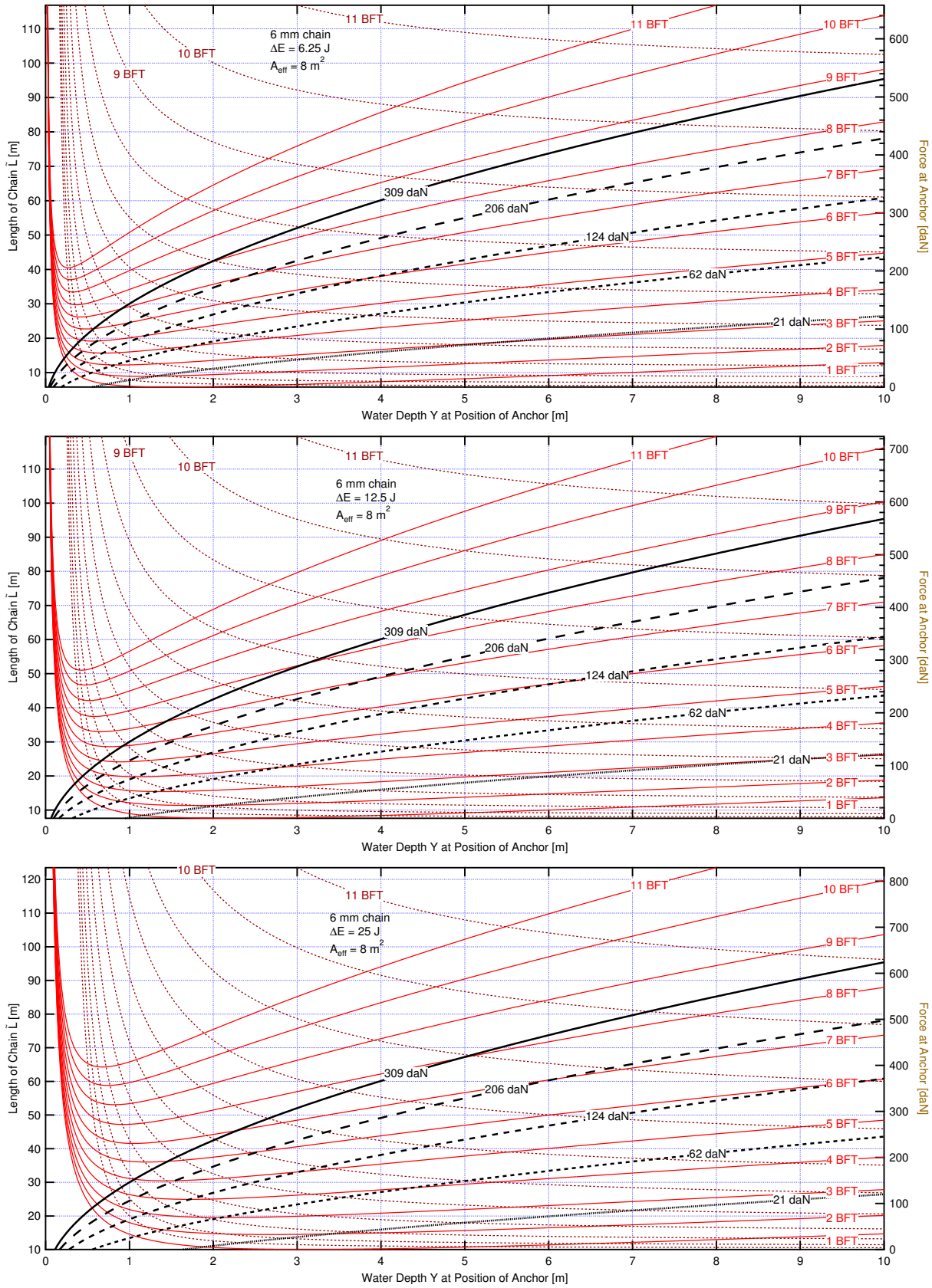

Fig. 2. Zoom of Fig. 1.

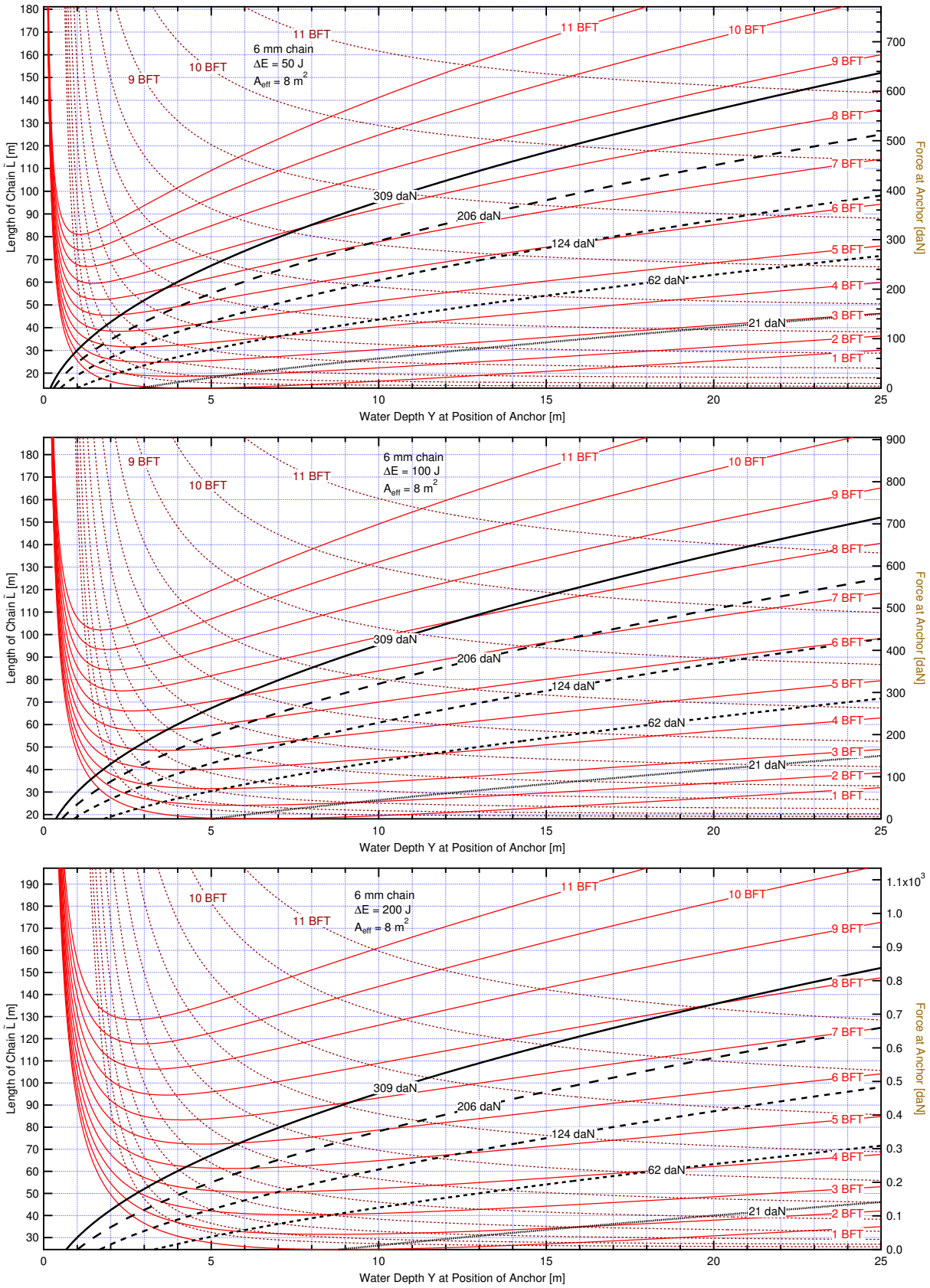

Fig. 3. Required chain length $\tilde{L}$ as function of anchor depth Y and wind strength for 6 mm chain with $A_{\text{eff}} = 8\,\text{m}^2$, $\Delta E = 50\,\text{J}$, $100\,\text{J}$, $200\,\text{J}$. Anchor load (brown) to the right. Black lines represent various *assumed* limits to anchor load.

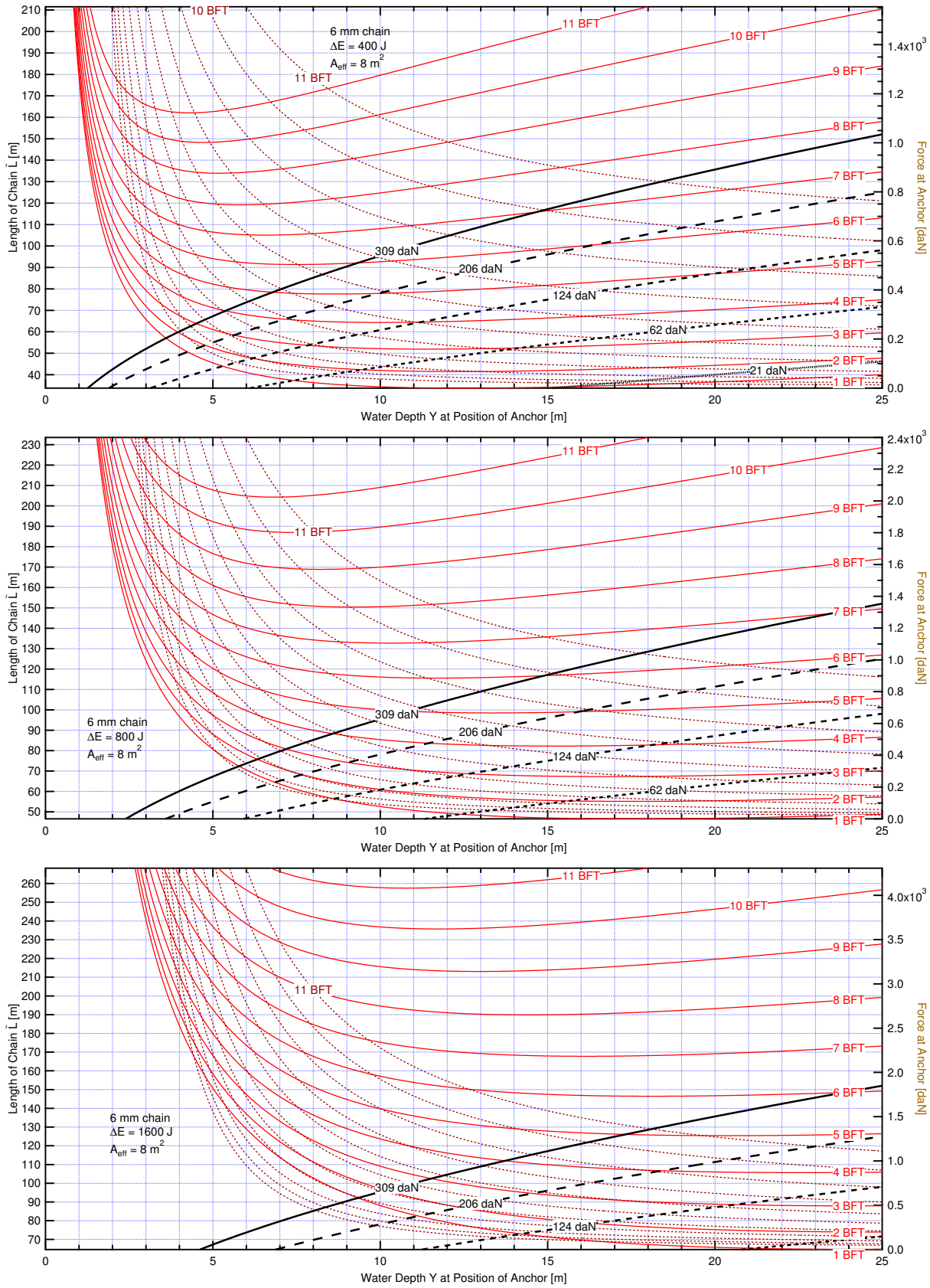

Fig. 4. Required chain length $\tilde{L}$ as function of anchor depth Y and wind strength for 6 mm chain with $A_{\text{eff}} = 8\,\text{m}^2$, $\Delta E = 400\,\text{J}$, $800\,\text{J}$, $1600\,\text{J}$. Anchor load (brown) to the right. Black lines represent various *assumed* limits to anchor load:.

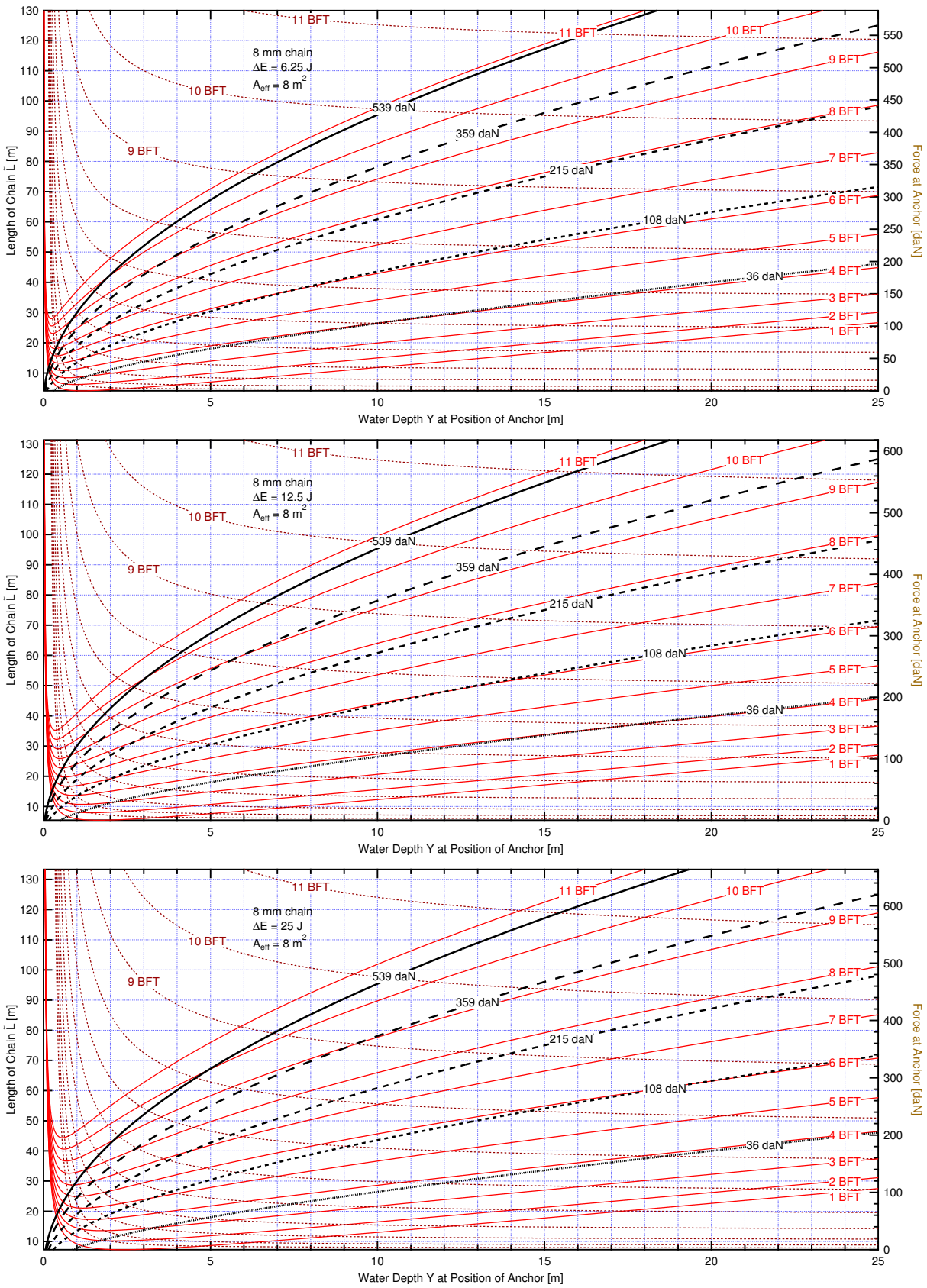

Fig. 5. Required chain length $\tilde{L}$ as function of anchor depth Y and wind strength for 8 mm chain with $A_{\text{eff}} = 8\,\text{m}^2$, $\Delta E = 6.25\,\text{J}$, $12.5\,\text{J}$, $25\,\text{J}$. Anchor load (brown) to the right. Black lines represent various *assumed* limits to anchor load.

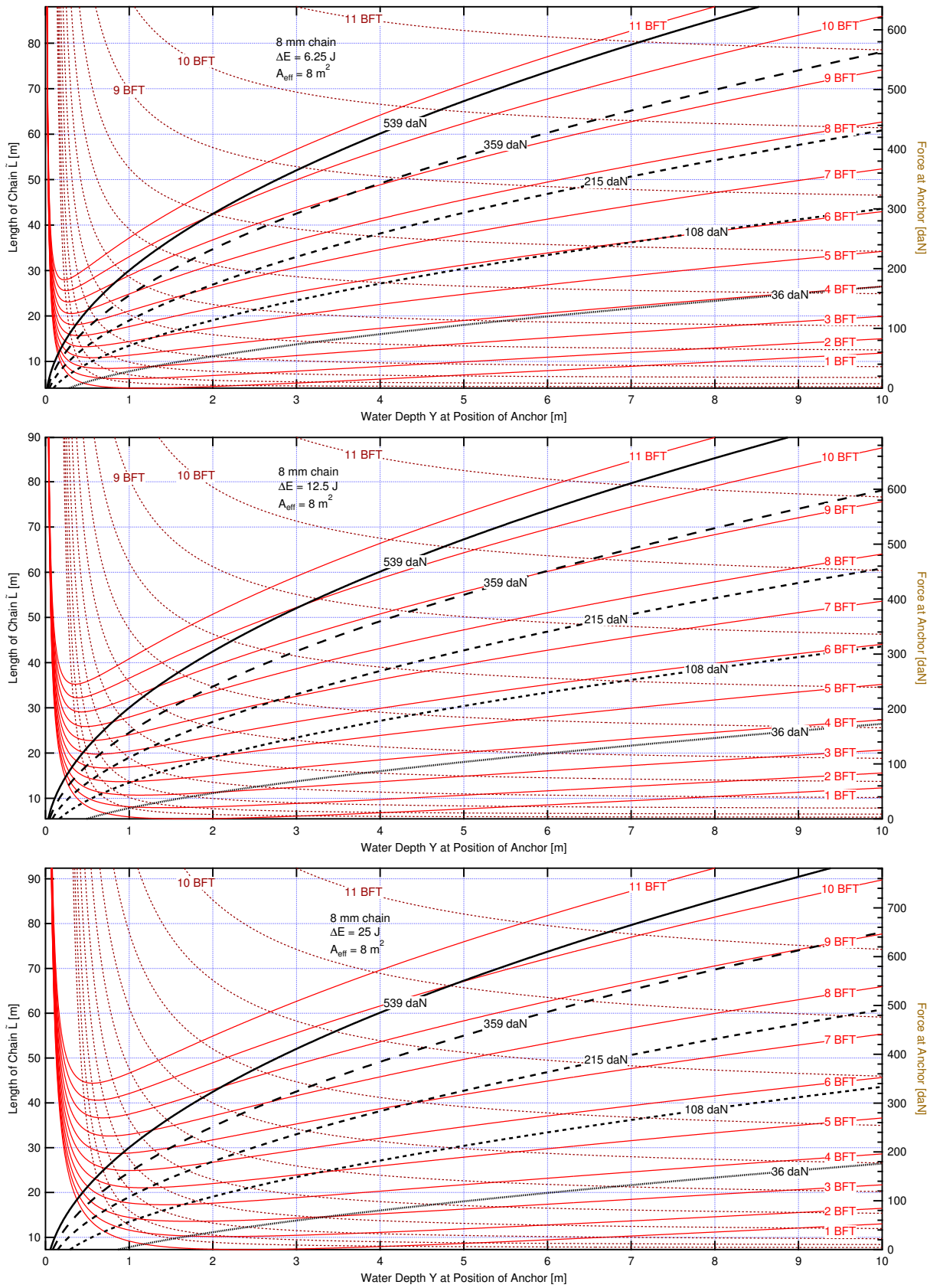

Fig. 6. Zoom of Fig. 5.

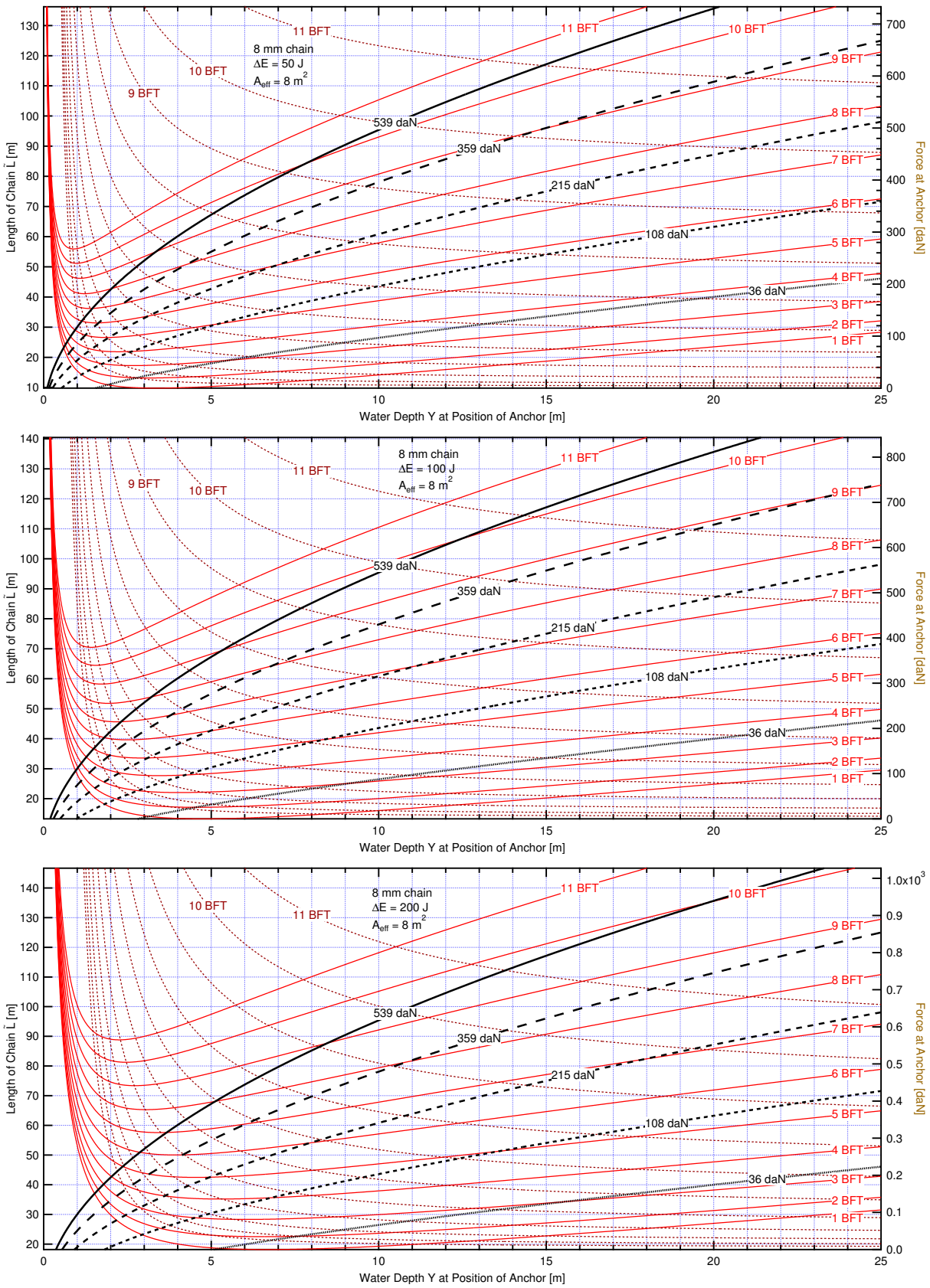

Fig. 7. Required chain length $\tilde{L}$ as function of anchor depth Y and wind strength for 8 mm chain with $A_{\text{eff}} = 8\,\text{m}^2$, $\Delta E = 50\,\text{J}$, $100\,\text{J}$, $200\,\text{J}$. Anchor load (brown) to the right. Black lines represent various *assumed* limits to anchor load.

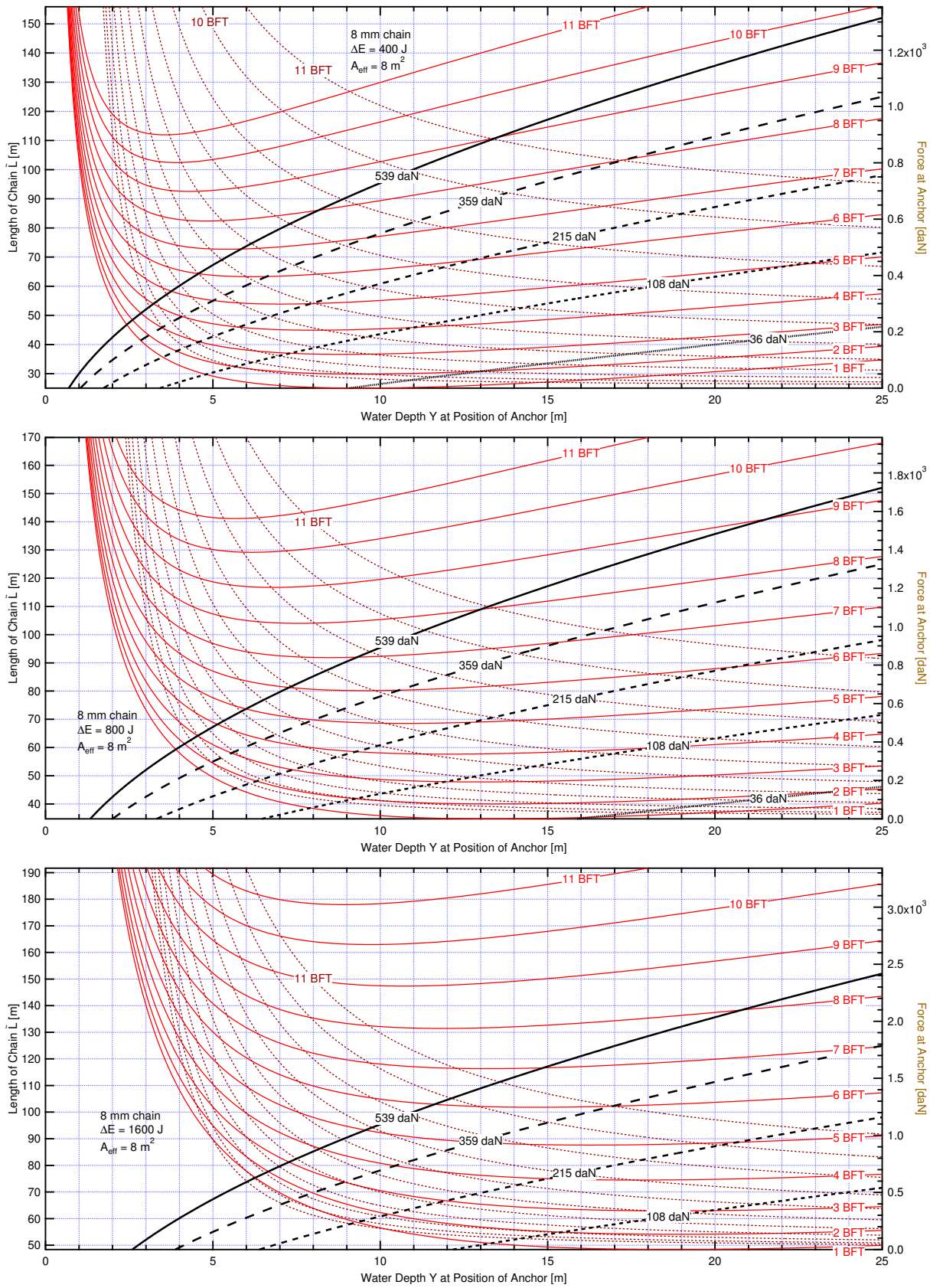

Fig. 8. Required chain length $\tilde{L}$ as function of anchor depth Y and wind strength for 8 mm chain with $A_{\text{eff}} = 8\,\text{m}^2$, $\Delta E = 400\,\text{J}$, $800\,\text{J}$, $1600\,\text{J}$. Anchor load (brown) to the right. Black lines represent various *assumed* limits to anchor load.

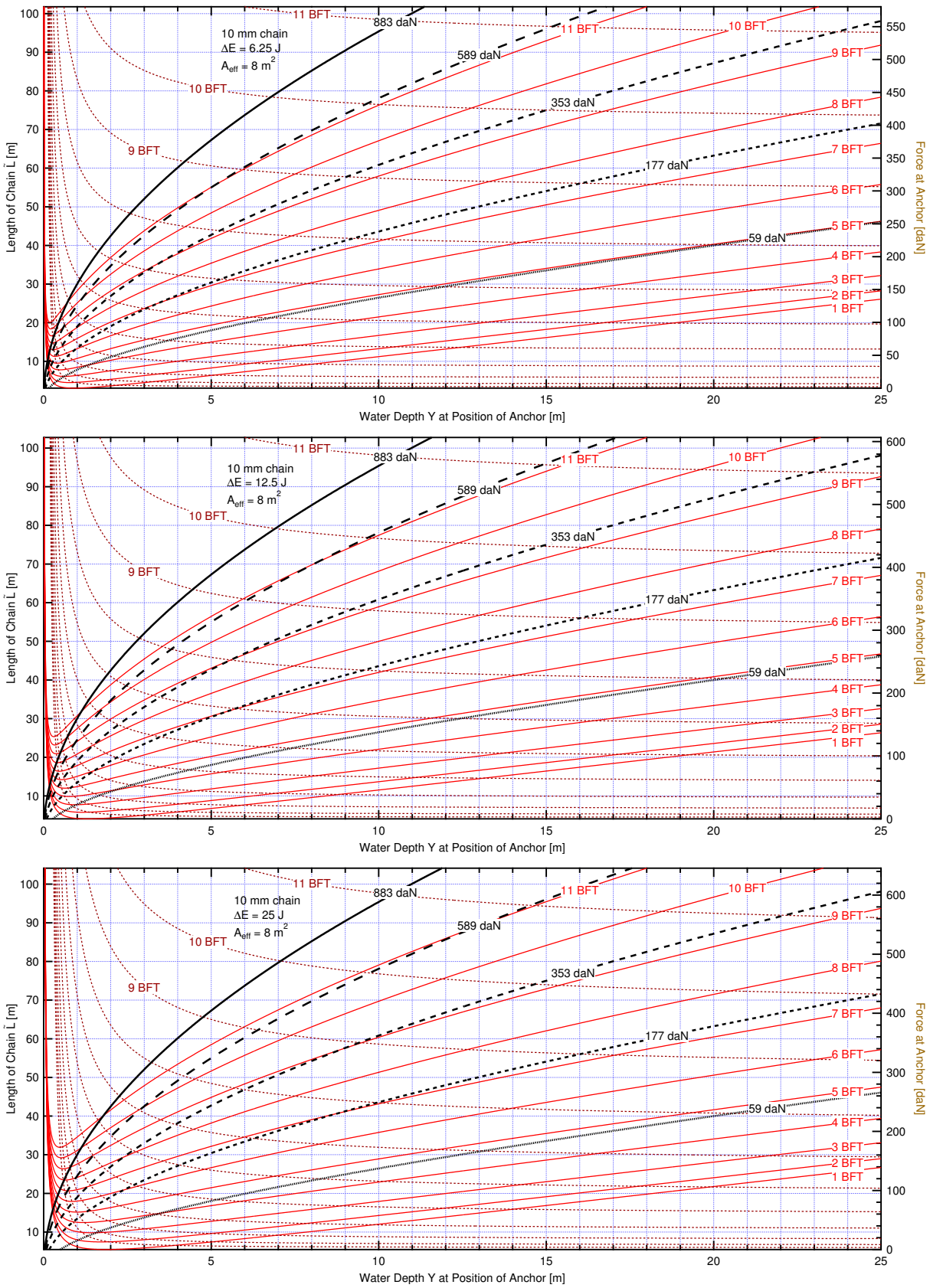

Fig. 9. Required chain length $\tilde{L}$ as function of anchor depth Y and wind strength for 10 mm chain with $A_{\text{eff}} = 8\,\text{m}^2$, $\Delta E = 6.25\,\text{J}$, $12.5\,\text{J}$, $25\,\text{J}$. Anchor load (brown) to the right. Black lines represent various *assumed* limits to anchor load.

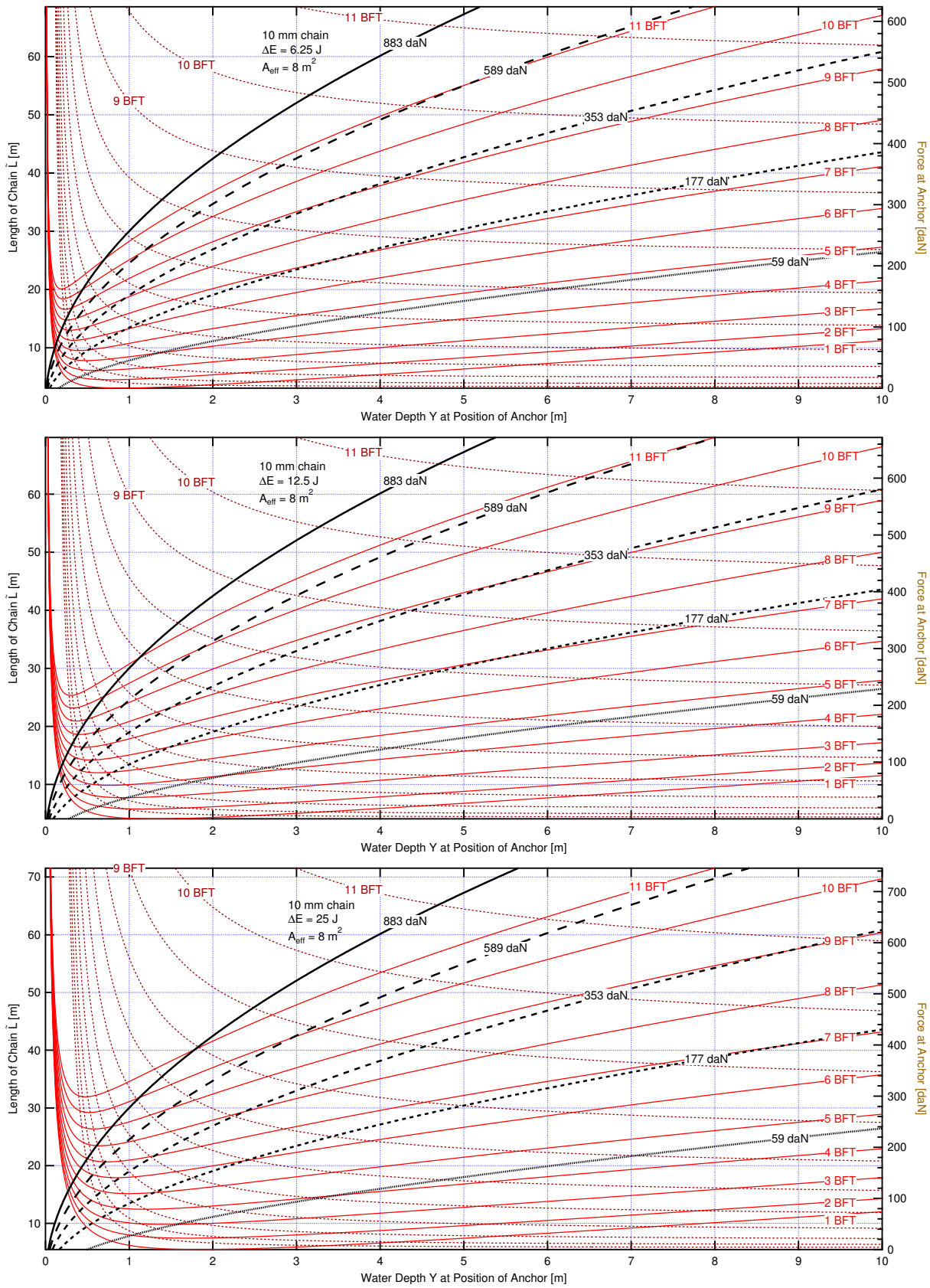

Fig. 10. Zoom of Fig. 9.

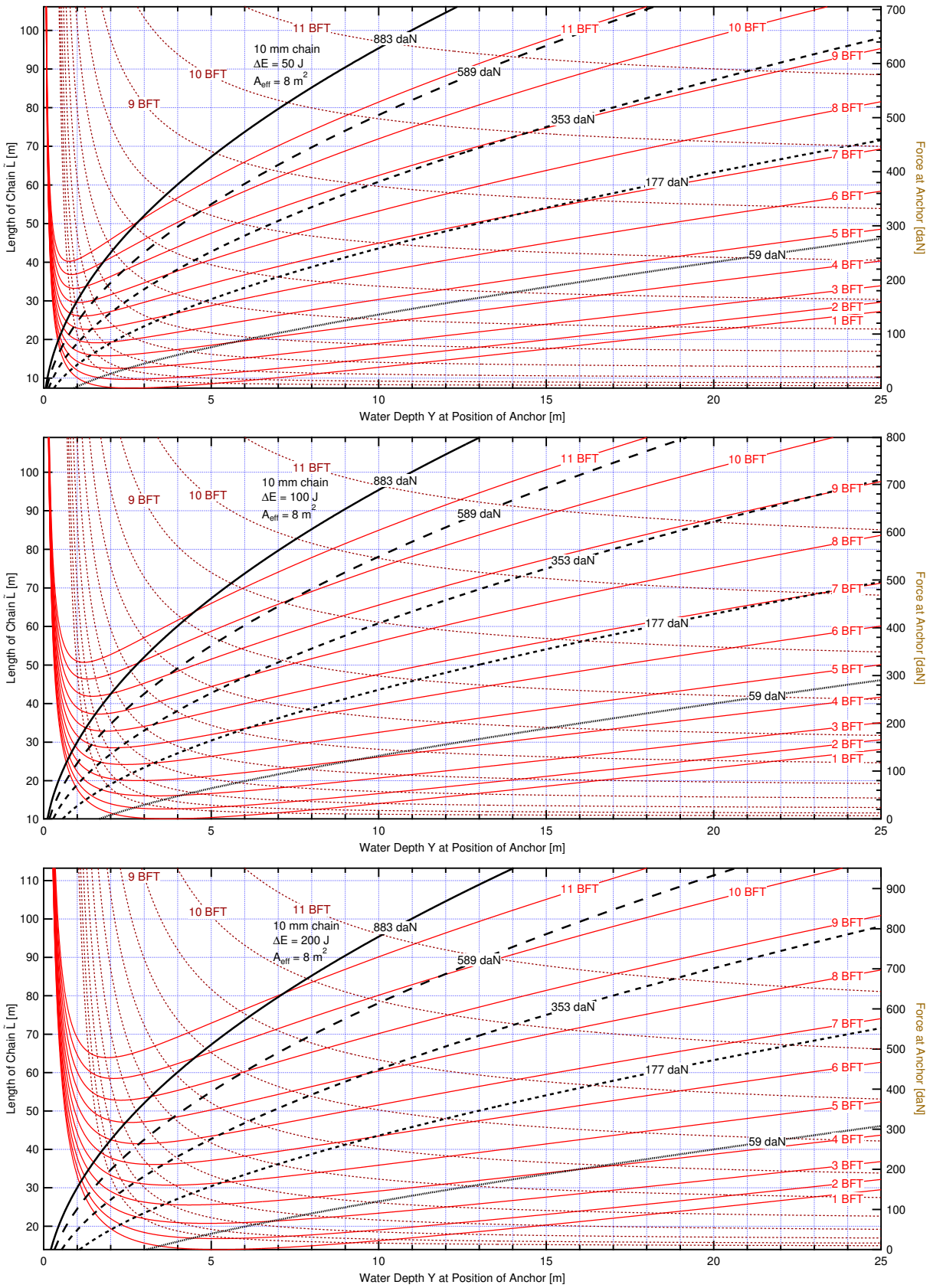

Fig. 11. Required chain length $\tilde{L}$ as function of anchor depth Y and wind strength for 10 mm chain with $A_{\text{eff}} = 8\,\text{m}^2$, $\Delta E = 50\,\text{J}$, $100\,\text{J}$, $200\,\text{J}$. Anchor load (brown) to the right. Black lines represent various *assumed* limits to anchor load.

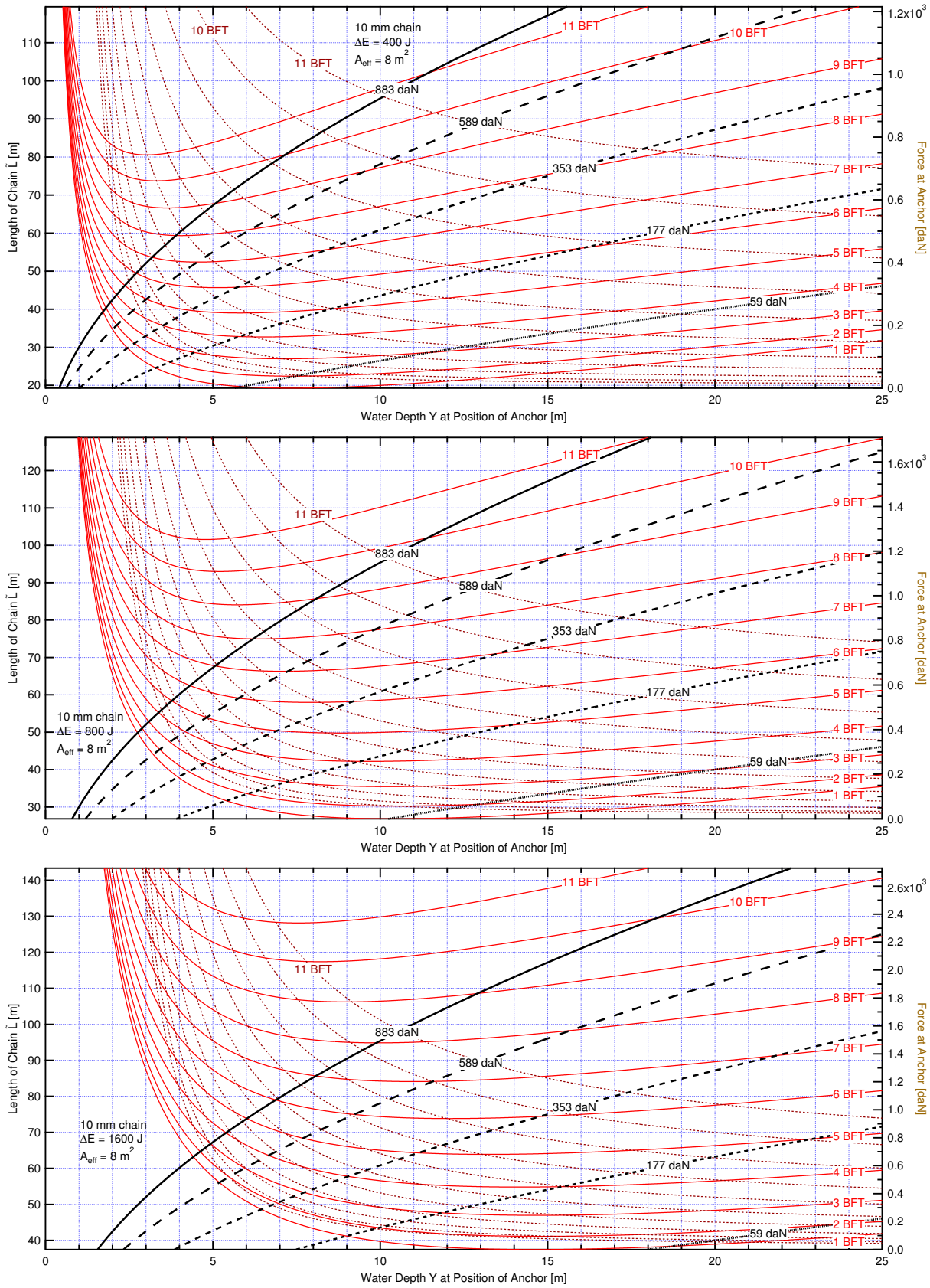

Fig. 12. Required chain length $\tilde{L}$ as function of anchor depth Y and wind strength for 10 mm chain with $A_{\text{eff}} = 8\,\text{m}^2$, $\Delta E = 400\,\text{J}$, $800\,\text{J}$, $1600\,\text{J}$. Anchor load (brown) to the right. Black lines represent various *assumed* limits to anchor load.

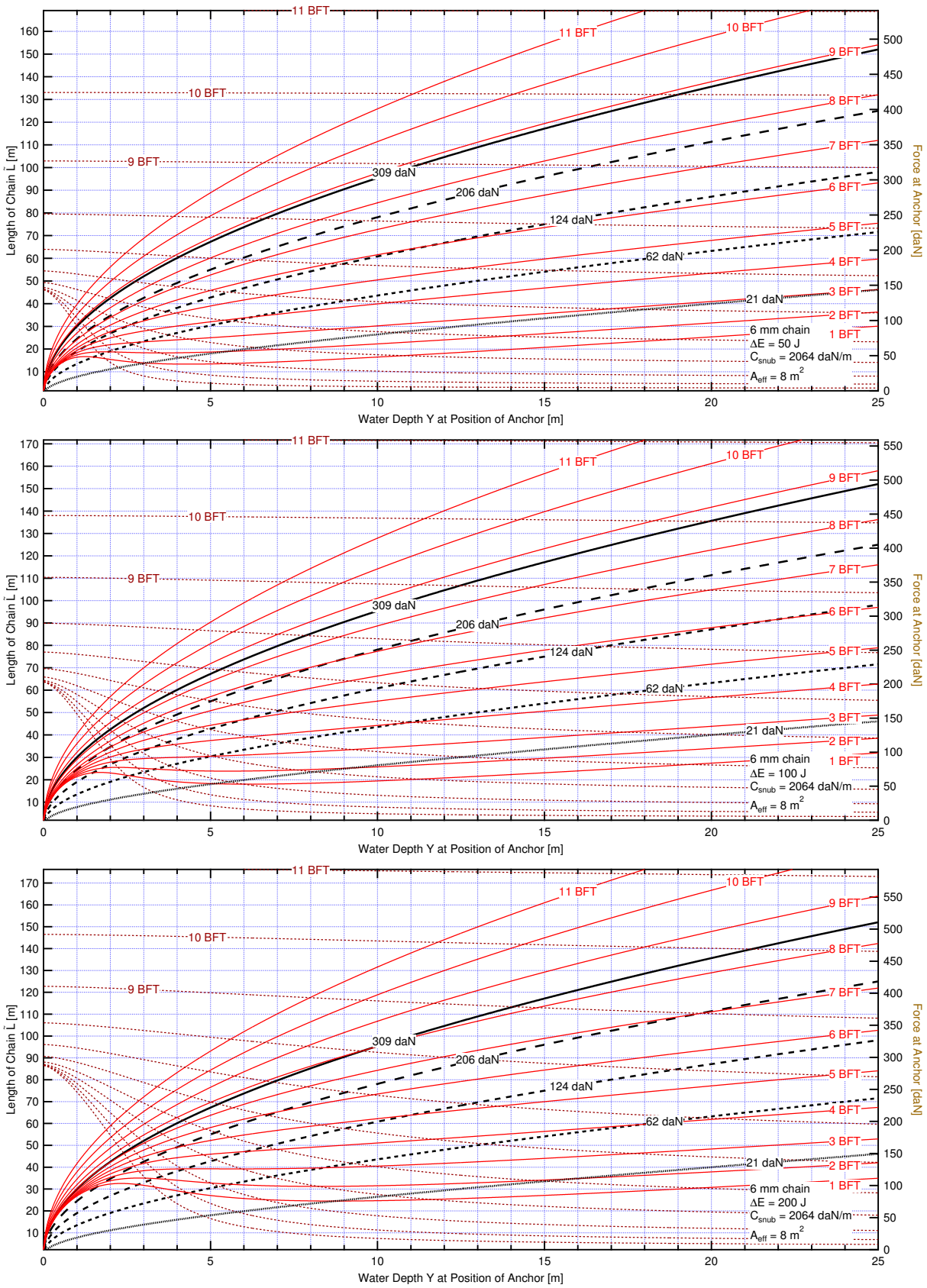

Fig. 13. Same as Fig. 3, but now with a snubber that elongates by 10 cm at 8 BFT and $\Delta E = 0$. Note that for $\Delta E > 0$ this elongation can easily be a multiple of this! E.g., for $Y = 0$: $\delta x = \sqrt{2C\Delta E + F_W^2}/C$.

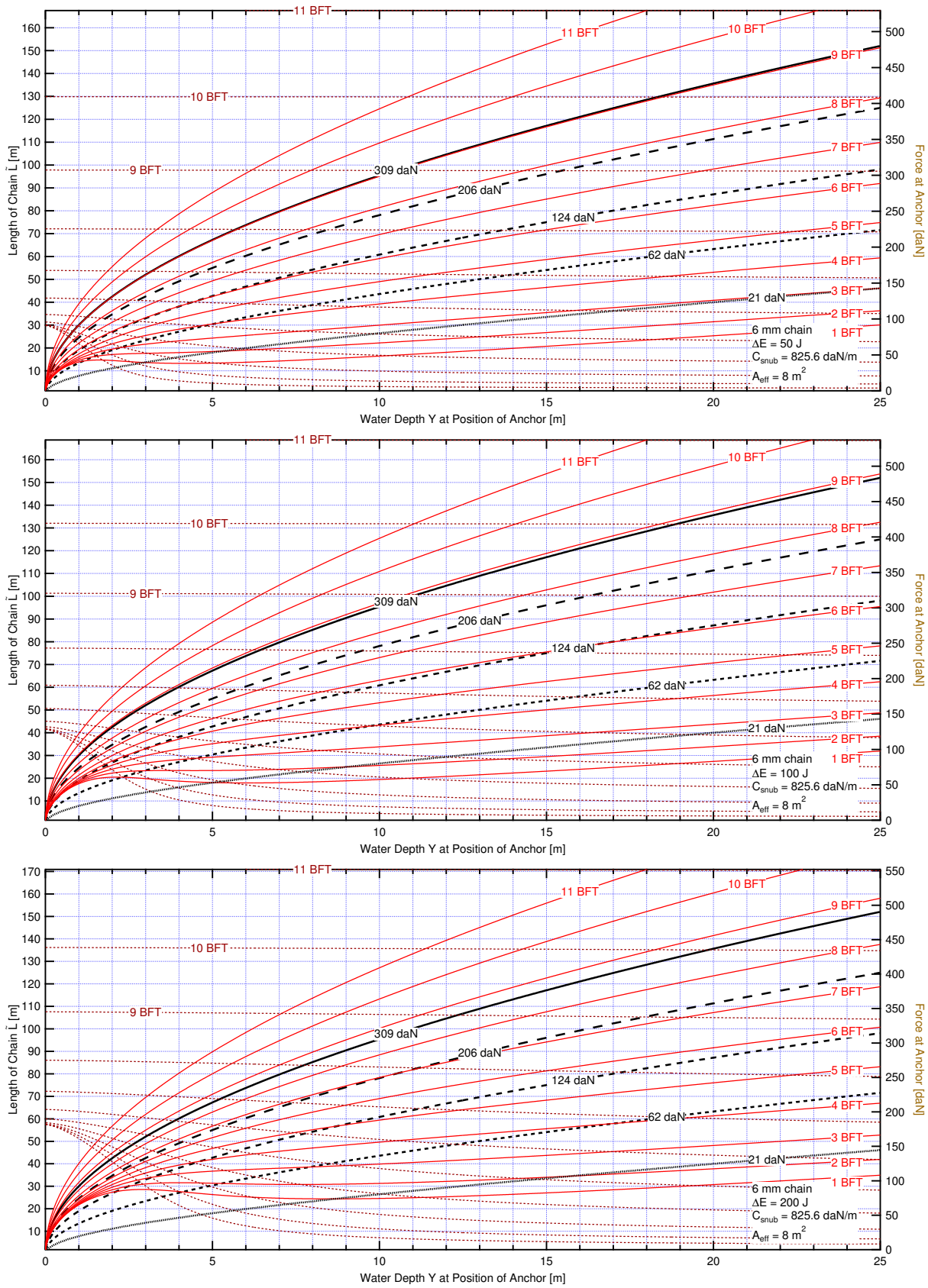

Fig. 14. Same as Fig. 3, but now with a snubber that elongates by 25 cm at 8 BFT and $\Delta E = 0$. Note that for $\Delta E > 0$ this elongation can easily be a multiple of this! E.g., for $Y = 0$: $\delta x = \sqrt{2C\Delta E + F_W^2}/C$.

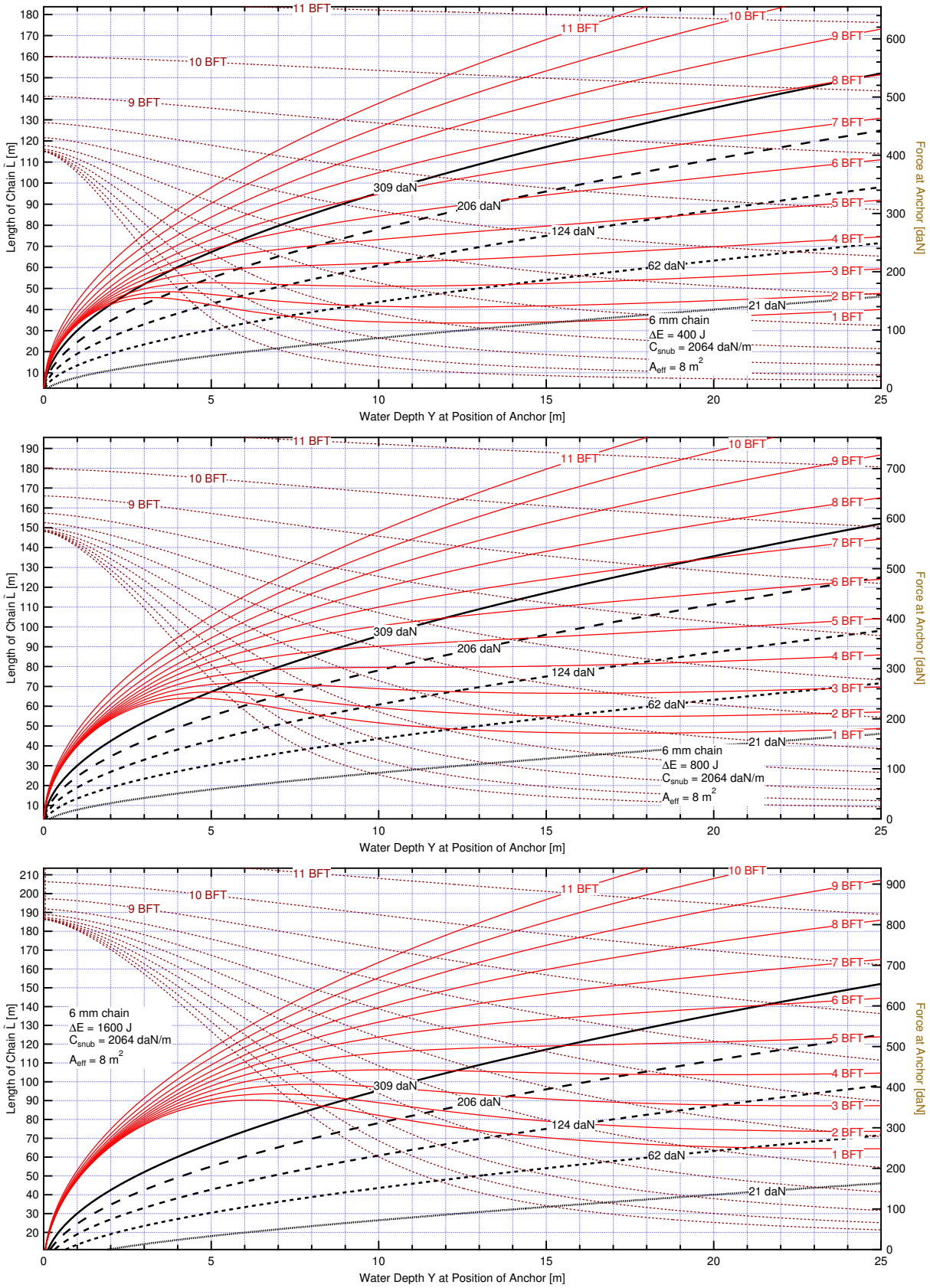

Fig. 15. Same as Fig. 4, but now with a snubber that elongates by 10 cm at 8 BFT and $\Delta E = 0$. Note that for $\Delta E > 0$ this elongation can easily be a multiple of this! E.g., for $Y = 0$: $\delta x = \sqrt{2C\Delta E + F_W^2}/C$.

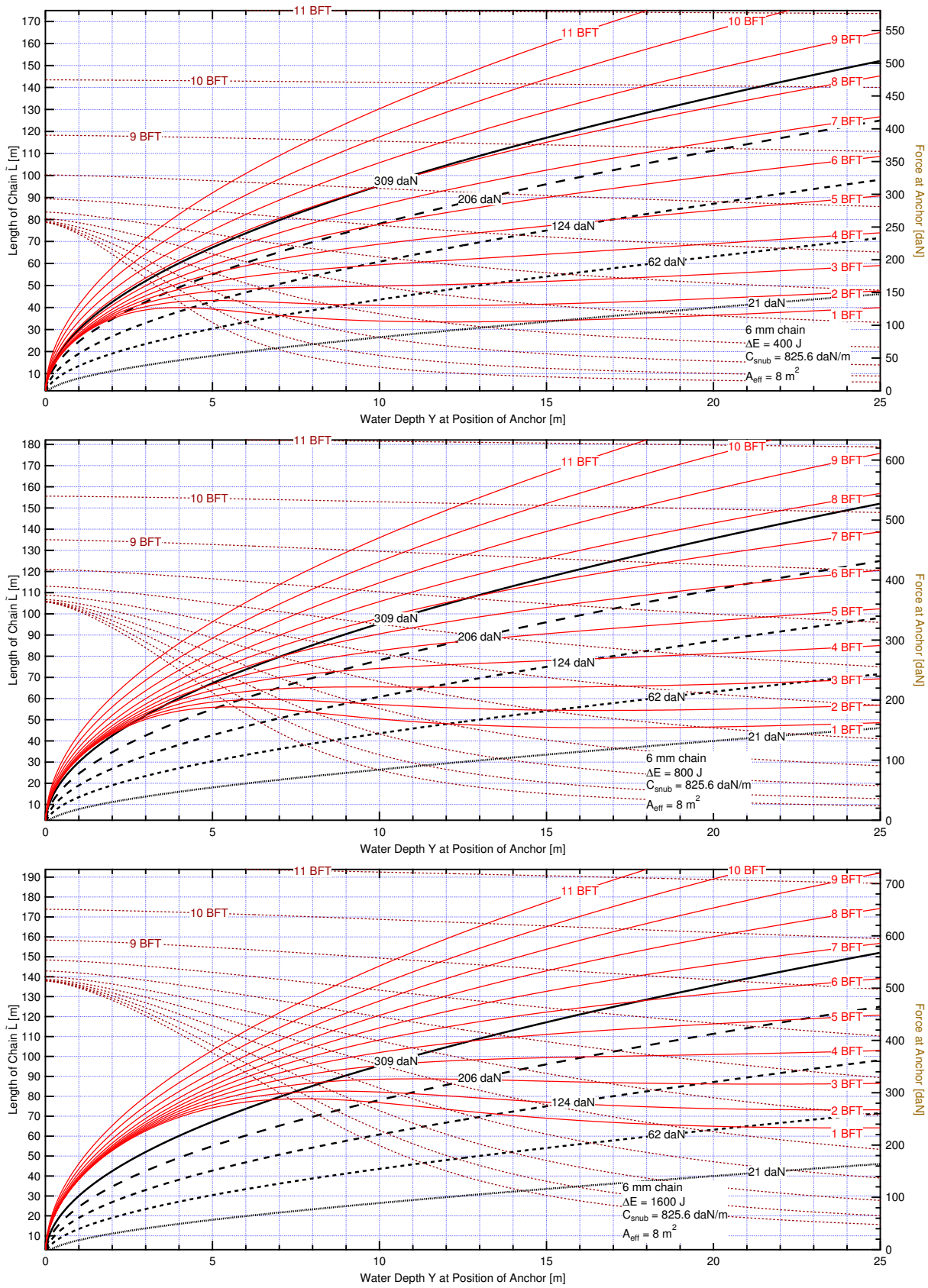

Fig. 16. Same as Fig. 4, but now with a snubber that elongates by 25 cm at 8 BFT and $\Delta E = 0$. Note that for $\Delta E > 0$ this elongation can easily be a multiple of this! E.g., for $Y = 0$: $\delta x = \sqrt{2C\Delta E + F_W^2}/C$.

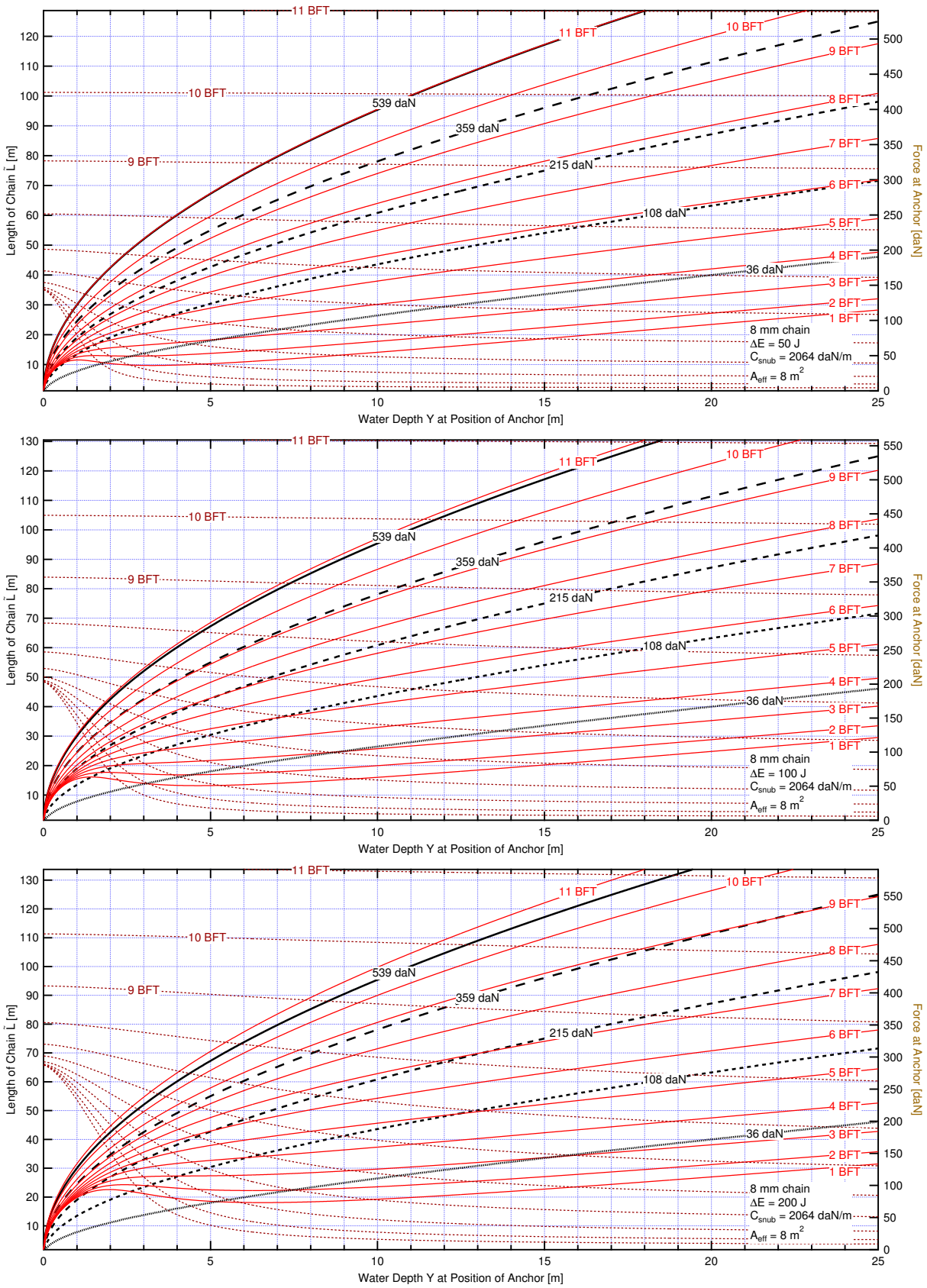

Fig. 17. Same as Fig. 7, but now with a snubber that elongates by 10 cm at 8 BFT and $\Delta E = 0$. Note that for $\Delta E > 0$ this elongation can easily be a multiple of this! E.g., for $Y = 0$: $\delta x = \sqrt{2C\Delta E + F_W^2}/C$.

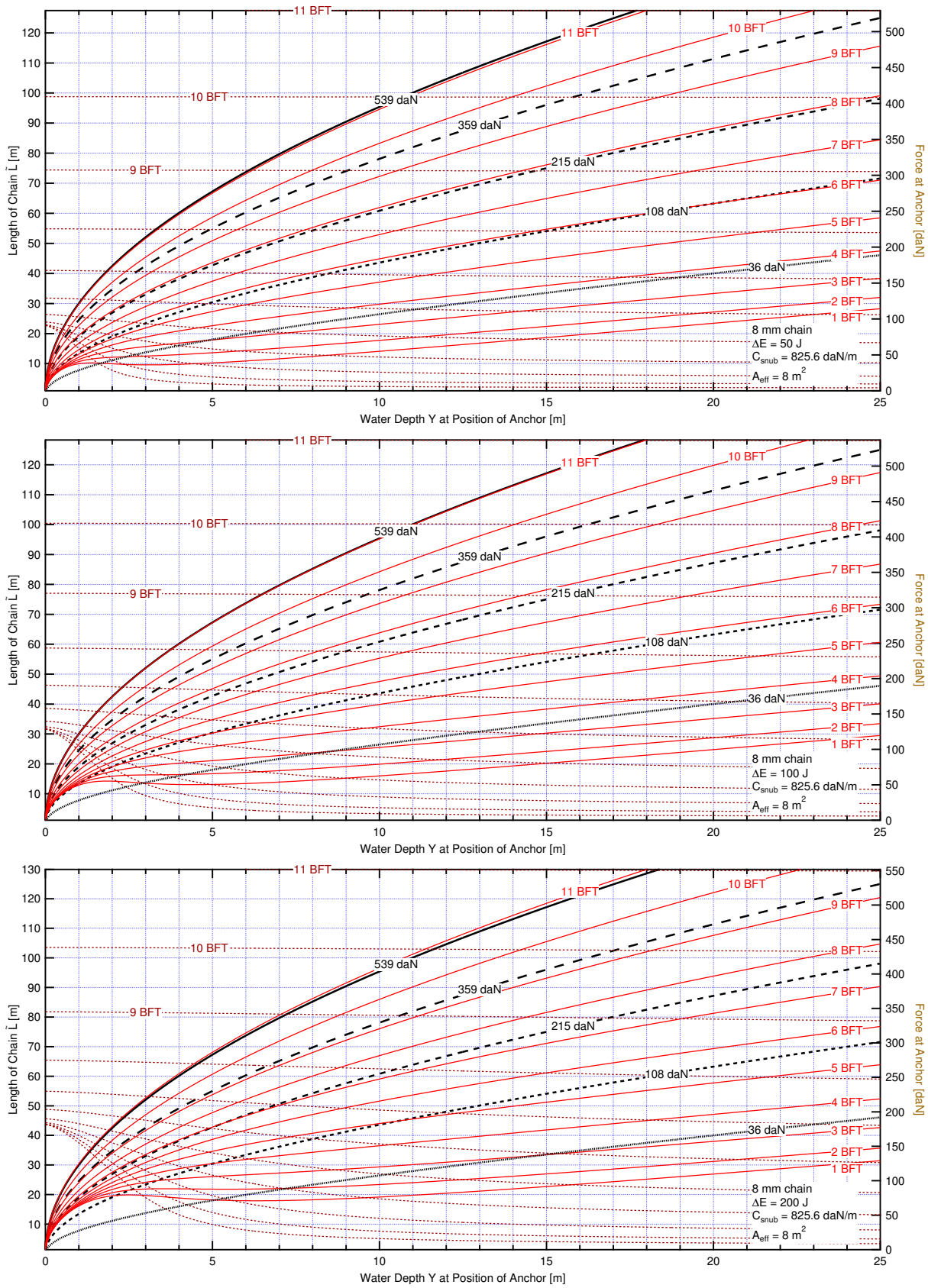

Fig. 18. Same as Fig. 7, but now with a snubber that elongates by 25 cm at 8 BFT and $\Delta E = 0$. Note that for $\Delta E > 0$ this elongation can easily be a multiple of this! E.g., for $Y = 0$: $\delta x = \sqrt{2C\Delta E + F_W^2}/C$.

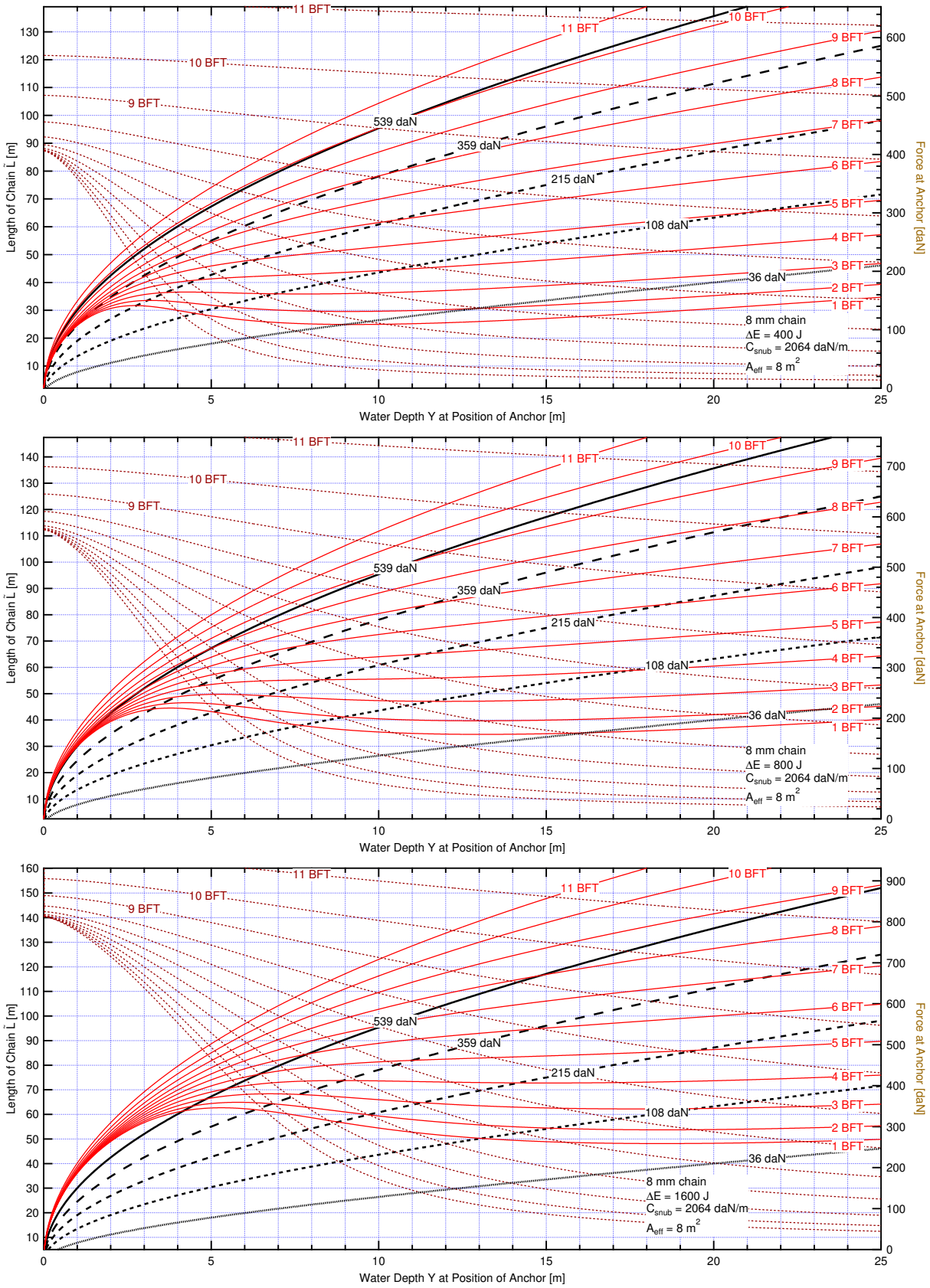

Fig. 19. Same as Fig. 8, but now with a snubber that elongates by 10 cm at 8 BFT and $\Delta E = 0$. Note that for $\Delta E > 0$ this elongation can easily be a multiple of this! E.g., for $Y = 0$: $\delta x = \sqrt{2C\Delta E + F_W^2}/C$.

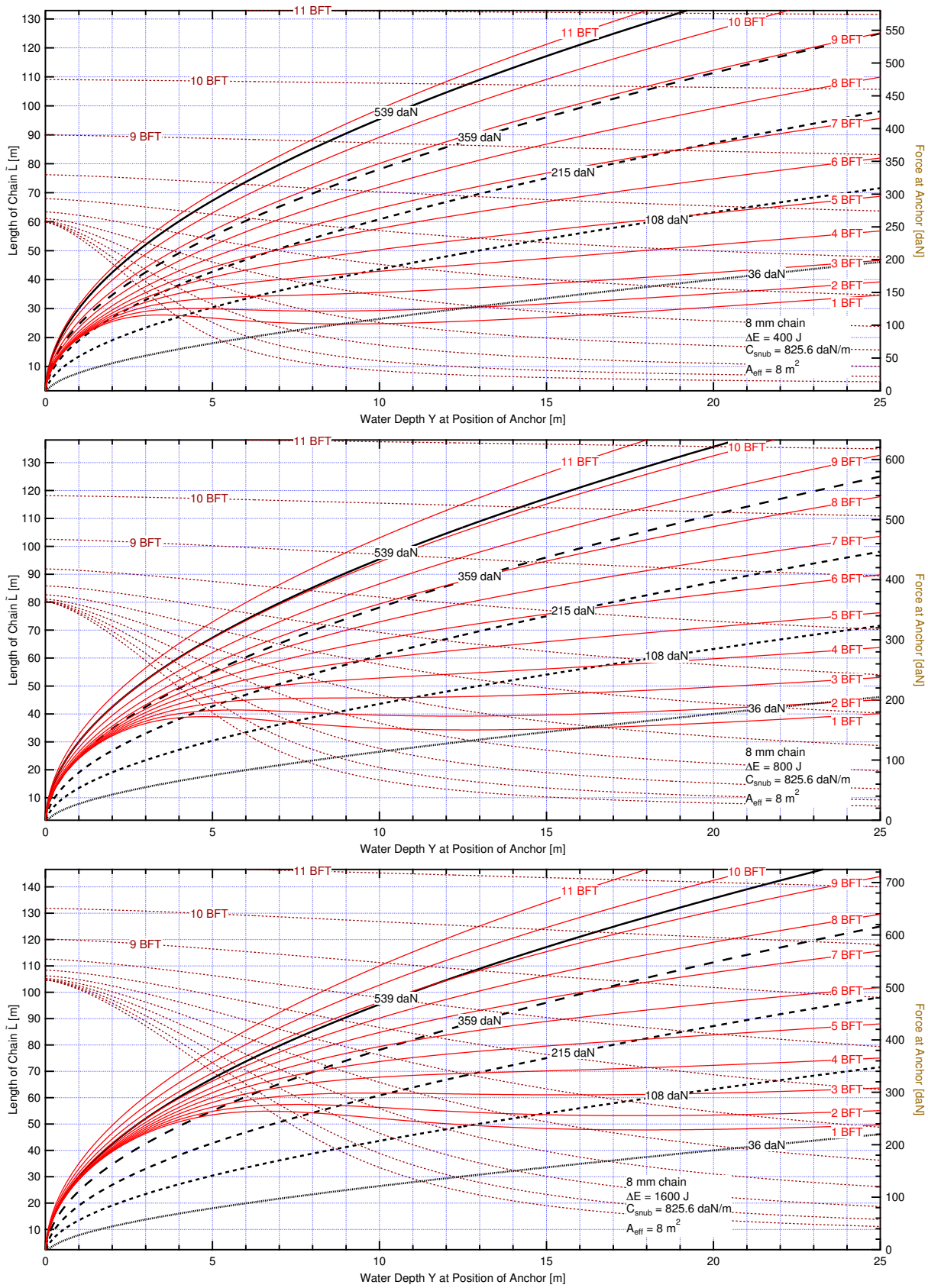

Fig. 20. Same as Fig. 8, but now with a snubber that elongates by 25 cm at 8 BFT and $\Delta E = 0$. Note that for $\Delta E > 0$ this elongation can easily be a multiple of this! E.g., for $Y = 0$: $\delta x = \sqrt{2C\Delta E + F_W^2}/C$.

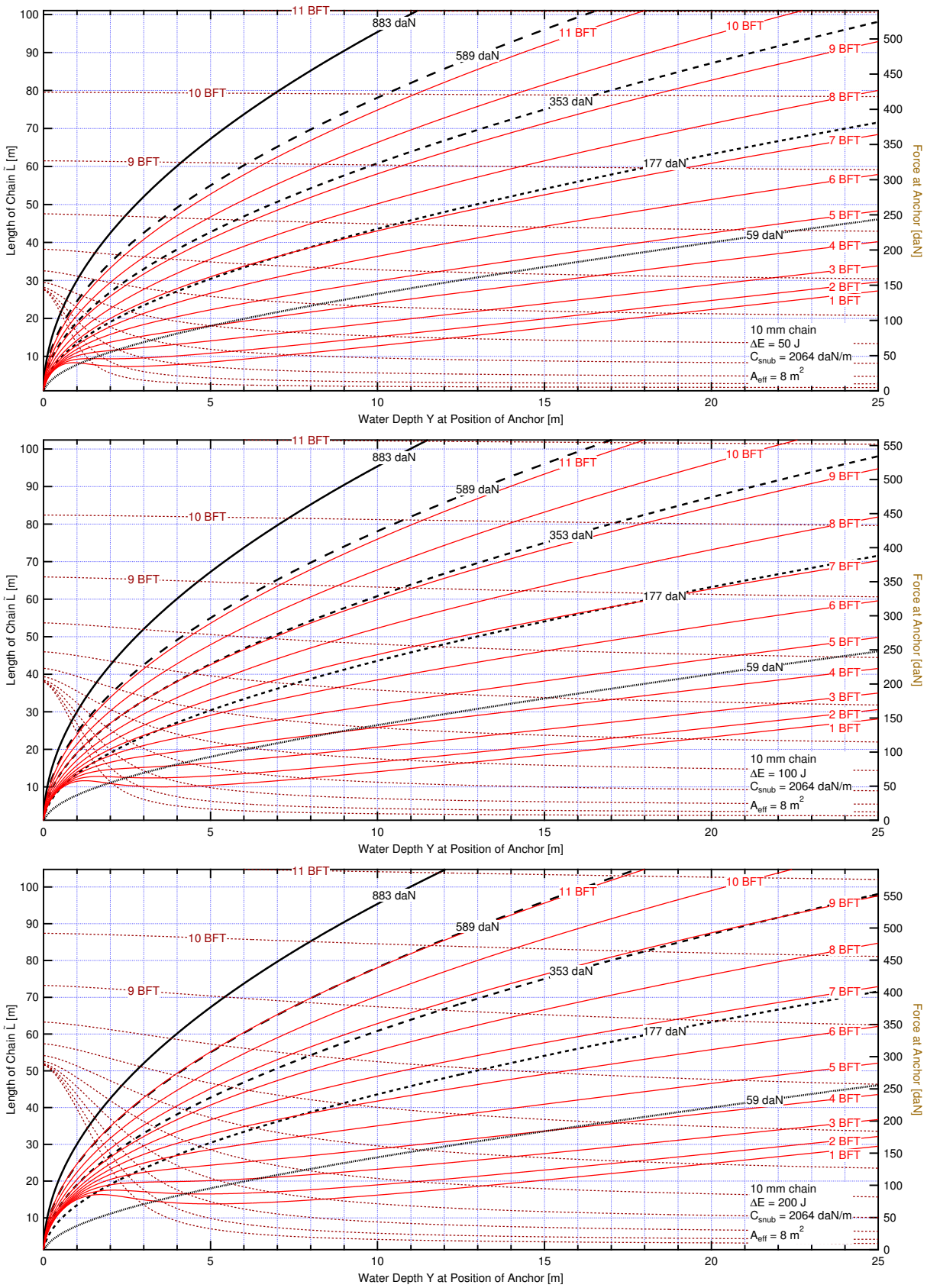

Fig. 21. Same as Fig. 11, but now with a snubber that elongates by 10 cm at 8 BFT and $\Delta E = 0$. Note that for $\Delta E > 0$ this elongation can easily be a multiple of this! E.g., for $Y = 0$: $\delta x = \sqrt{2C\Delta E + F_W^2}/C$.

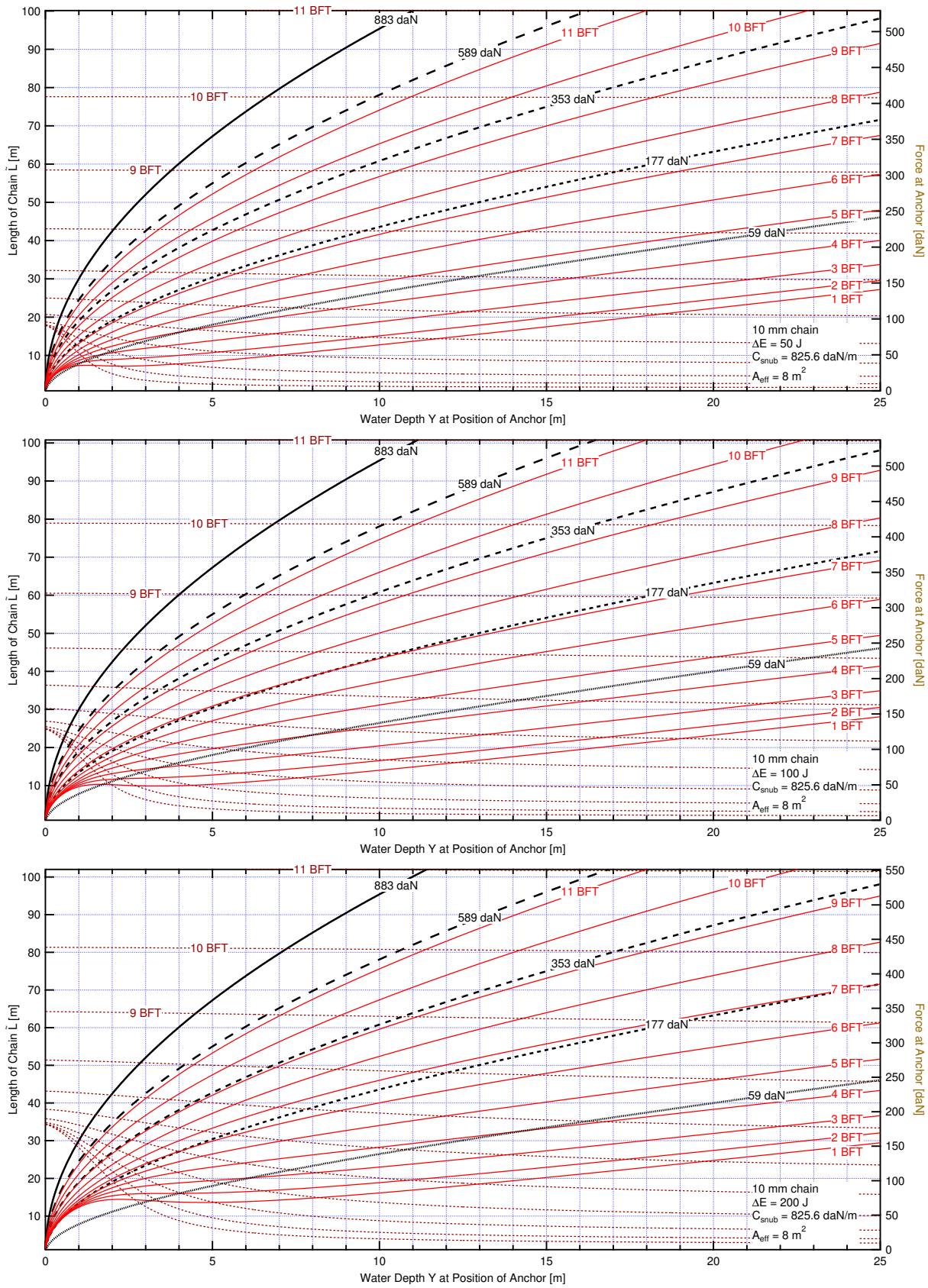

Fig. 22. Same as Fig. 11, but now with a snubber that elongates by 25 cm at 8 BFT and $\Delta E = 0$. Note that for $\Delta E > 0$ this elongation can easily be a multiple of this! E.g., for $Y = 0$: $\delta x = \sqrt{2C\Delta E + F_W^2}/C$.

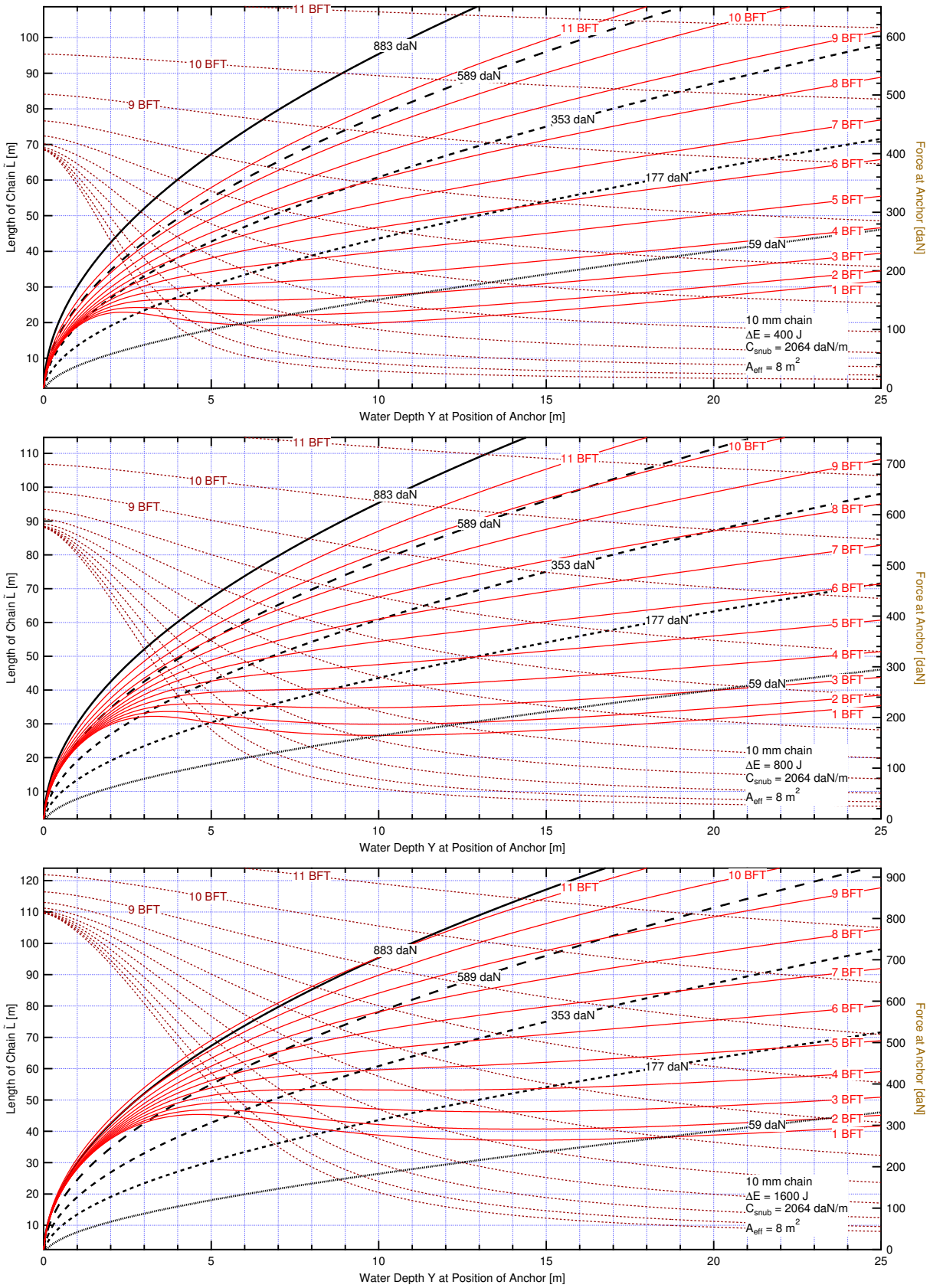

Fig. 23. Same as Fig. 12, but now with a snubber that elongates by 10 cm at 8 BFT and $\Delta E = 0$. Note that for $\Delta E > 0$ this elongation can easily be a multiple of this! E.g., for $Y = 0$: $\delta x = \sqrt{2C\Delta E + F_W^2}/C$.

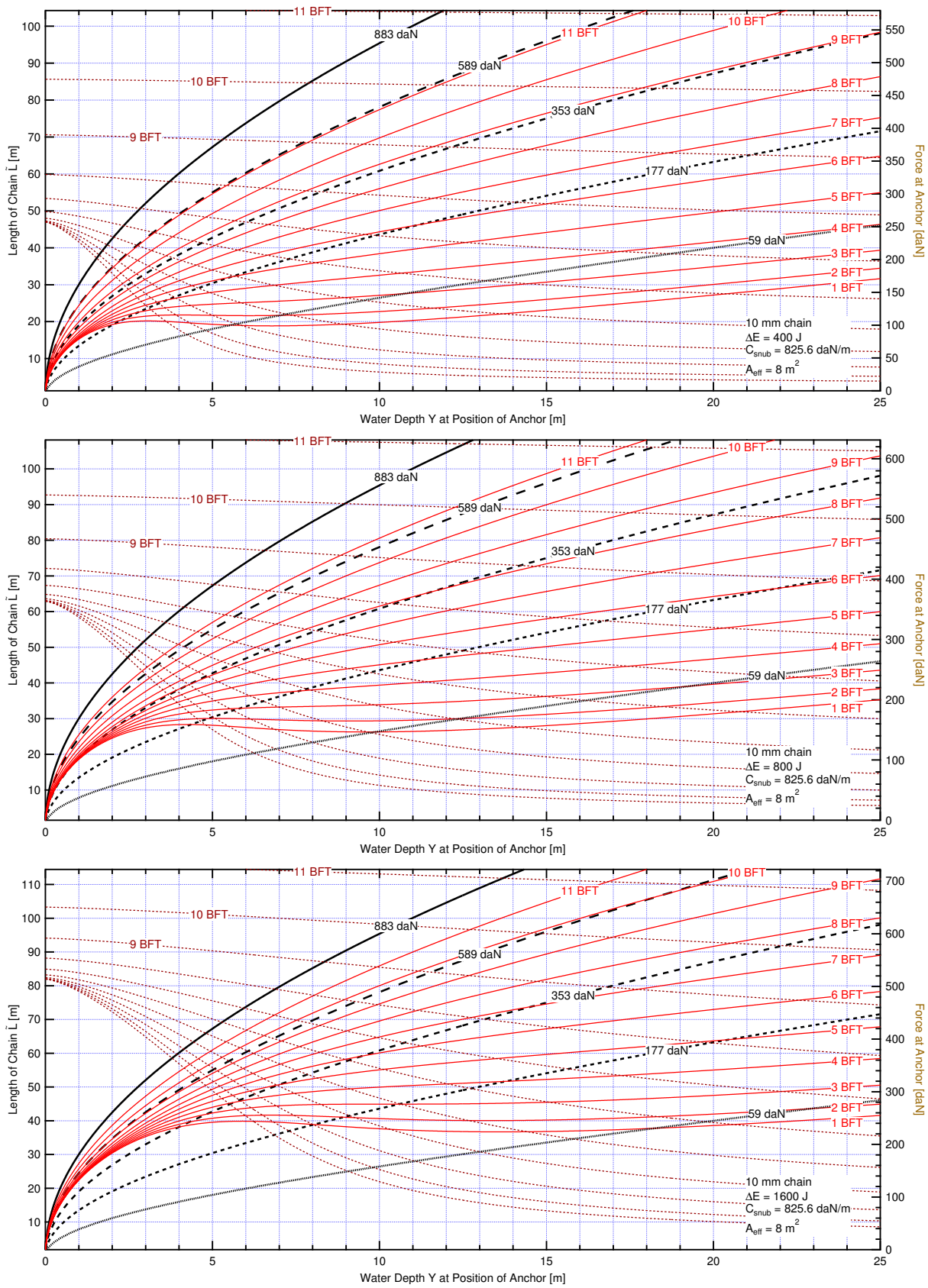

Fig. 24. Same as Fig. 12, but now with a snubber that elongates by 25 cm at 8 BFT and $\Delta E = 0$. Note that for $\Delta E > 0$ this elongation can easily be a multiple of this! E.g., for $Y = 0$: $\delta x = \sqrt{2C\Delta E + F_W^2}/C$.